# Network Analysis and Synthesis.

## UNIT-I.

· Network Topology.

**Network.**

It is the collection of interconnected components.

**Network Analysis.**

The process of finding voltage drop and current passing through each and every component of the network is said to be network Analysis.

In order to analyse the network the techniques called nodal analysis, mesh analysis, star to delta transformation . . . etc can be used.

But if the network complexity increases then above listed techniques will become more complex.

Under this case, a technique called Network Topology is used to analyse the complex networks.

## NETWORK TOPOLOGY

It is the study of properties of a network by investigating interconnected branches and nodes when R, L, C elements are replaced by a line (or) curve with two dots at its end.

Network Topology is also called as Graph Theory (or) Graph of Network.

In network Topology geometrical properties are being considered to analyse the Network.

23/09/2023

While representing network with its graph of network, the following.

1) All the network elements except the sources are replaced by a line or a curve.

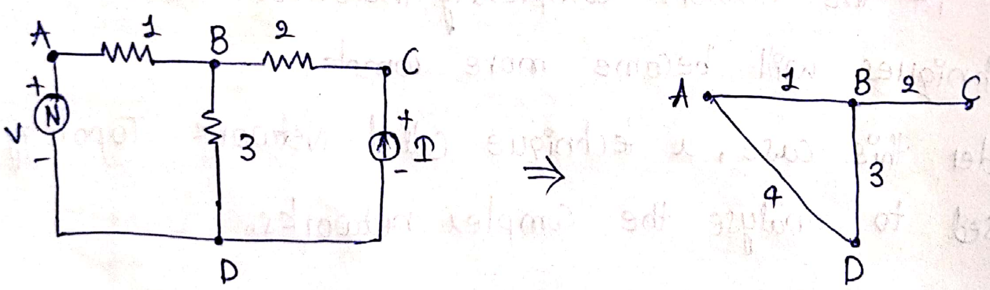
2) The voltage source is replaced with a short circuit and current source is replaced with open circuit.

3) All the nodes are represented with dot.

⇒ A Graph is represented with
1) Dot
2) St line
3) Curve.

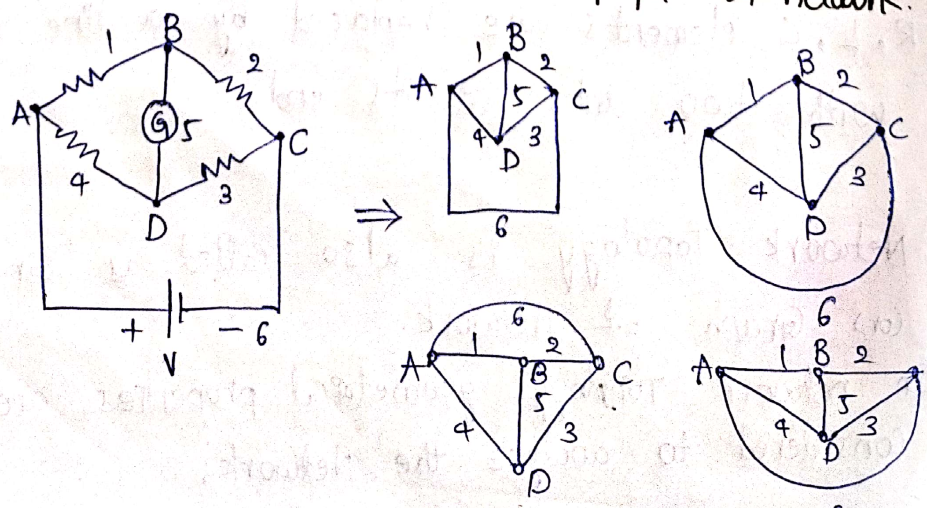
Ex-1 :-

Network                Graph of network.

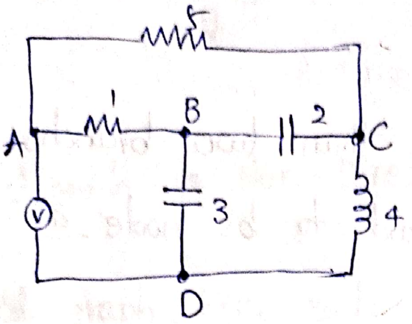



Ex-2 :-

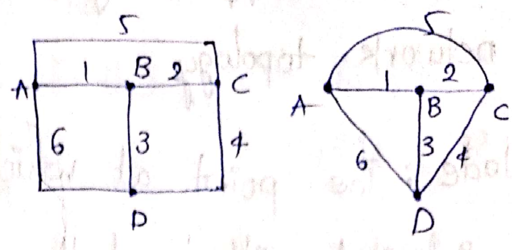
Network                Graph of network.



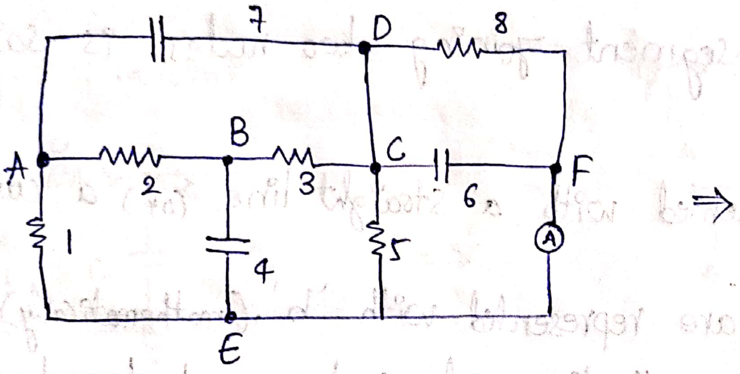
Ex-3 :- Network



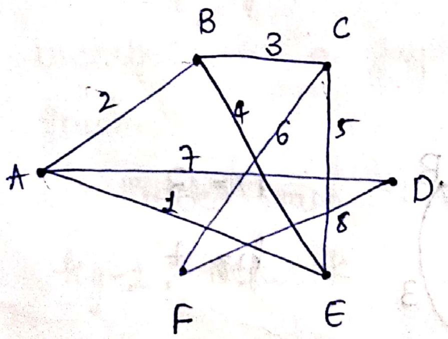

Graph of network.

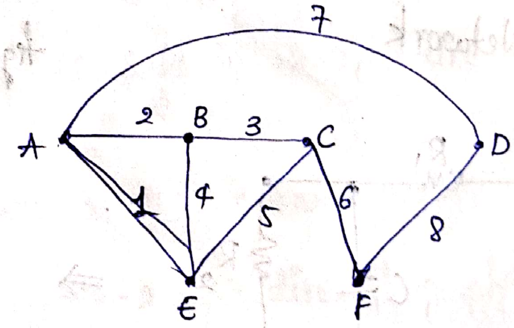


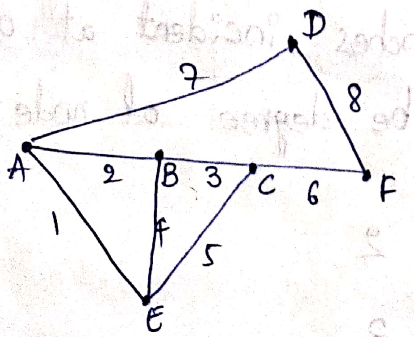

Ex- 4 :- Network.

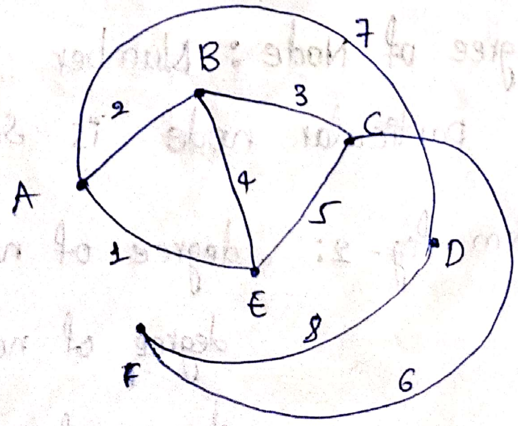



Graph of network.



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Basic terminology of graph theory (or) graph of network (or) network topology

**Node :** The point at which more than two branches intersect with eachother, is said to be node. It is represented with dot (.).

Mathematical representation.

Mathematically no. of nodes are represented with 'n'.

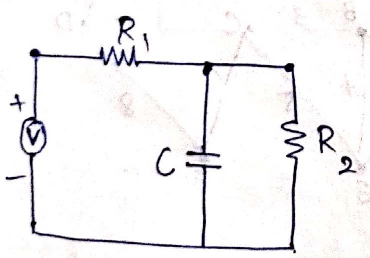
**Branch :** The line segment joining two nodes is said to be branch.

Branch is represented with a straight line (or) a curve.

No. of branches are represented with 'b'. (mathematically).

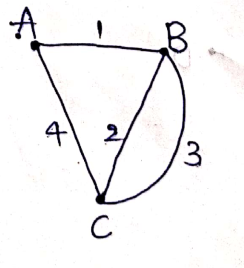
**Graph :** It is the collection of nodes and branches.

fig-1: Network




⇒

fig-2: Graph




$n = 3$

$b = 4$

**Degree of Node :** Number of branches incident at one particular node is said to be degree of node.

from fig-2: degree of node 'A' = 2

degree of node 'B' = 3

degree of node 'C' = 3

⇒ No. of branches connected to a particular node is said to be "degree of node."

Rank of the graph : If a graph has 'n' no. of nodes then the rank of the graph of the graph is : $R = n-1$

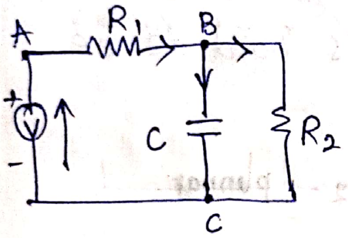
26/09/2023.

Directed graph & Non-Directed graph.

Directed graph :- If each and every branch of a graph has directions then the graph is said to be directed graph.

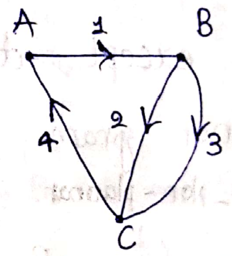
⇒ Directed graph is also called as oriented graph.

fig-1 : Network




⇒

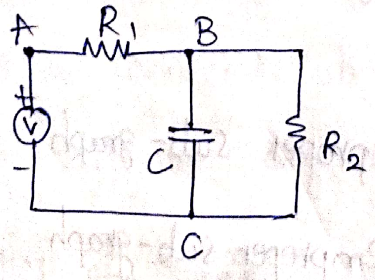
fig-2 : Directed graph




Un Directed graph :- If there are no directions for the branches in a graph. Then it is said to be Un-directed graph.

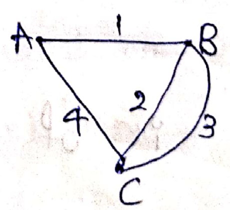
⇒ Un directed graph is also known as Un-oriented graph.

fig-1 :- Network




⇒

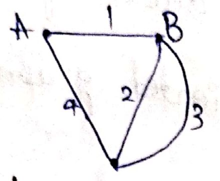
fig-2 :- Undirected graph.



# plannar and Non-plannar graphs.

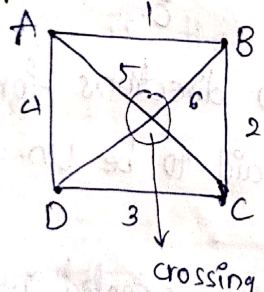
**plannar graph :-** If a graph Can be drawn in a plane surface in such a way that no two branches cross eachother except at nodes. Then it is Said to be plannar graph.

fig-1.



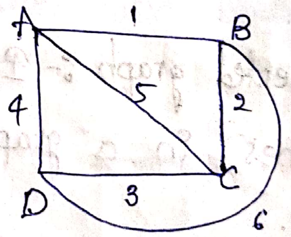

**Non-planar graph :-** If a graph Can be drawn in a plane surface in such a way that ~~no~~ two branches cross eachother except at nodes. Then it is said to be Non-planar graph.

fig-1 :- Non-plannar.




converted. ⇒

fig-2 - plannar.



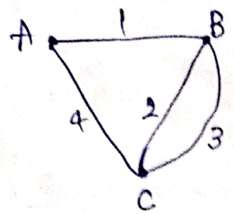

**Sub-graph.**

It is a Connection of sub-set of nodes and branches of a graph.

⇒ Sub-graph is of 2 types: 1) proper sub-graph.
2) Improper sub-graph.

**proper sub-graph :-** If a Sub-graph has less no. of nodes & branches then Original graph. Then it is Said to be proper sub-graph.

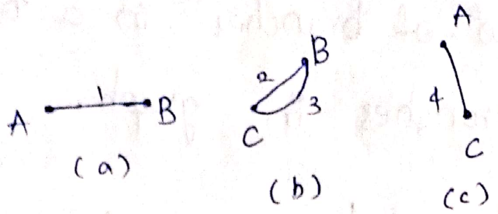
fig-1 :- graph




$n=3$

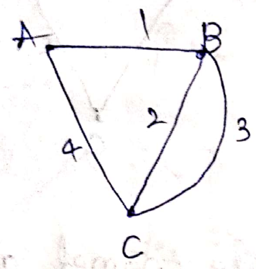
$b=4$

fig-2 : proper subgraph.




**Improper subgraph :-** If a sub-graph has equal no. of nodes when compared with original graph. then it is said to be Improper subgraph.

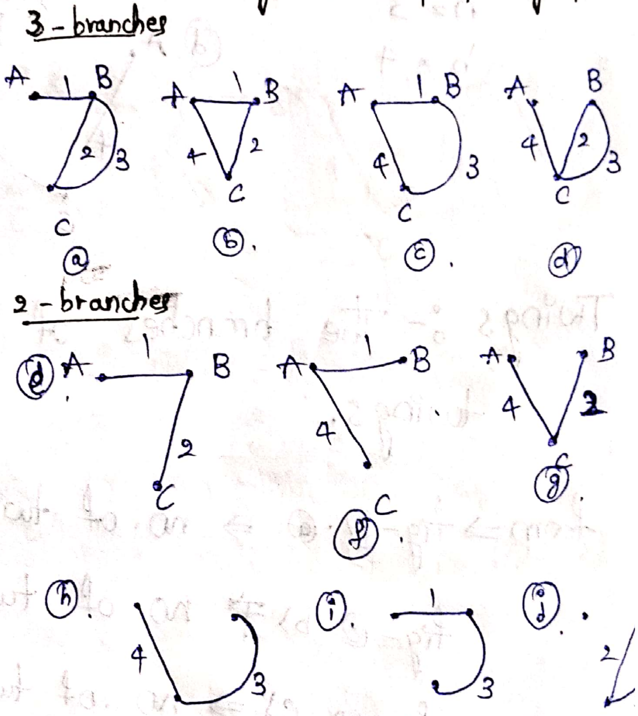
fig-1 :- graph




$n=3$

$b=4$

fig-2 :- Improper graph.




## Tree.

27/09/2023
wednesday

It is a connected sub-graph with equal no. of nodes and without any closed path (or) loop.

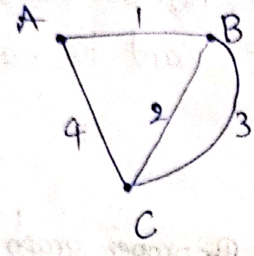
**Properties of a Trees:**

1. No. of nodes for a ~~grap~~ graph is equal to no. of nodes in a tree.
2. If $n$ is the no. of nodes of a tree, then it should contain $(n-1)$ branches.

③. (n-1) is known as rank of the tree.

④. no. of branches in a tree should be less than no. of branches in graph.

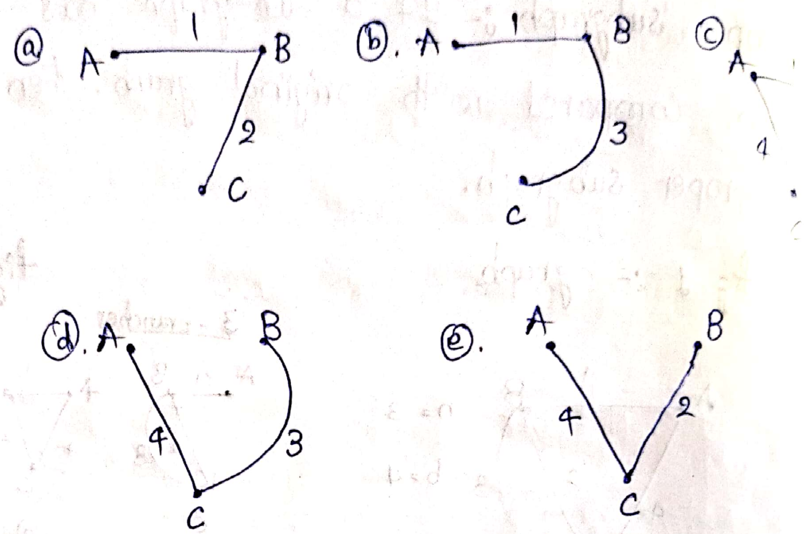
fig-1 :- graph




$n = 3$

$b = 4$

fig2:- Trees.




Twings :- The branches of a tree are named as twings.

from ⇒ fig-②·ⓐ. ⇒ no. of twings (T) = 2, {1, 2}

fig-② b) ⇒ no. of twings (T) = 2, {1, 3}

fig-②. c) ⇒ no. of twings (T) = 2, {1, 4}

fig-②. ⓓ. ⇒ no. of twings (T) = 2, {4, 3}

fig-②. ⓔ. ⇒ no. of twings (T) = 2, {4, 2}.

Links/chords :- In the construction of a tree few branches are to be removed (or) open, such branches are called as links/chords.

from ⇒ fig-②. -
ⓐ. no. of links (L) = 2, {3, 4}
ⓑ. no. of links (L) = 2, {2, 4}
ⓒ. no. of links (L) = 2, {2, 3}
ⓓ. no. of links (L) = 2, {1, 2}

⑥. no. of links (L) - 2, {1, 3}.

If b is no. of the branches of a graph, then the no. of links will be b-(n-1).

## Co-Tree

It is a complimentary set of branches of a tree.

⇒ Branches of Co-tree are called as links / chords.

⇒ Links are always represented with dotted lines.

fig-1 :- graph. fig-2 :- Trees.

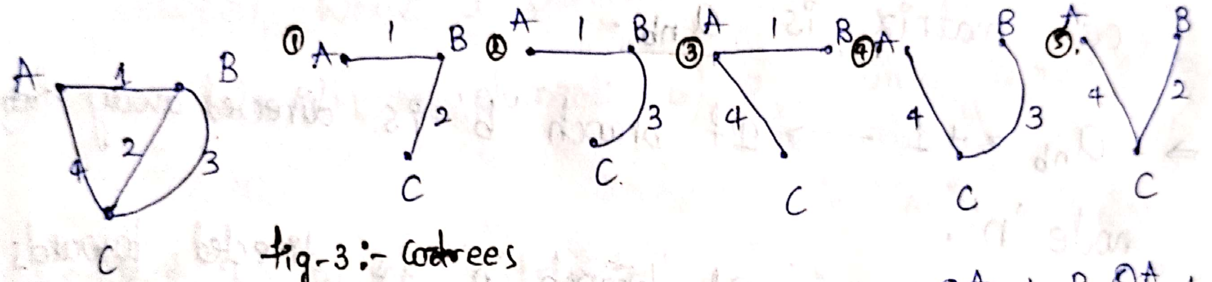
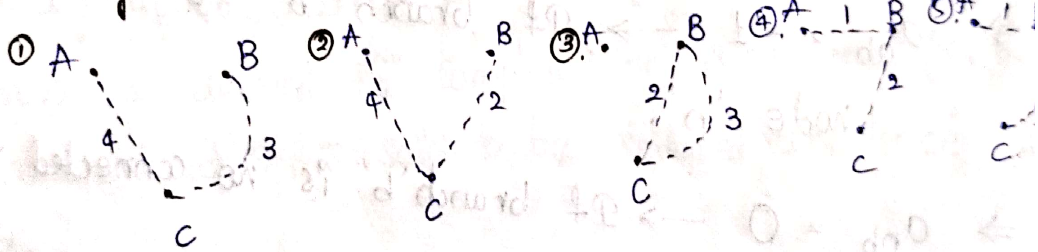


fig-3 :- Cotrees



## Incidence Matrix.

An oriented (or) directed graph can be mathematically represented in the form of a matrix, Called as Incidence matrix.

⇒ There are 2 types of Incidence matrices.

1) Complete Incidence Matrix.

2) Reduced Incidence Matrix.

Complete Incidence Matrix :- If a given network is Completely represented in the formed of a matrix, then it is named as Complete Incidence Matrix.

⇒ Complete Incidence Matrix is represented with $\underline{A}$ (or) ...

⇒ If 'n' represents no. of nodes & 'b' represents no. of branches, of a graph. Then the order of Complete Incidence matrix is $\underline{n \times b}$.

⇒ The standard convention for framing Complete incidence matrix is $\underline{a_{nb}}$.

⇒ $a_{nb} = +1 \longrightarrow$ If branch 'b' is directed away from node 'n'.

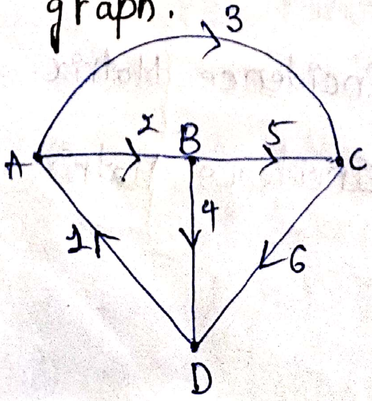
⇒ $a_{nb} = -1 \longrightarrow$ If branch 'b' is directed toward node 'n'.

⇒ $a_{nb} = 0 \longrightarrow$ If branch 'b' is no connected to node 'n'

⇒ no. of nodes 'n' represents rows.
no. of ~~nodes~~ branches 'b' represents coloumns.

Let us consider the following graph to frame Complete Incidence matrix.

fig :- graph.




$n = 4$
$b = 6$.

order $= n \times b$
$= 4 \times 6$.

$$A = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\!\!\begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 1 & 0 \\ 0 & 0 & -1 & 0 & -1 & 1 \\ 1 & 0 & 0 & -1 & 0 & -1 \end{bmatrix}_{4\times 6} \end{array}$$

nodes ↓ ; branches.

Properties of Complete Incidence Matrix.

1. Unit ~~en 'n' trees~~ entries $(\pm 1)$ in a row identifies the branches connected to a particular node.
2. Unit ~~en 'n' trees~~ entries in a coloum identify the nodes between which a particular branch is connected.
3. The sum of the elements of a coloumn should always be zero.

Reduced Incidence Matrix.

If a row is deleted from the complete incidence matrix, the the resultand matrix is said to be Reduced Incidence Matrix.

⇒ Reduced Incidence Matrix is represented with $A_r$ (or) $A_1$

⇒ If Reduced Incidence Matrix is provided, then it is possible to calculate no. of trees, for a given graph

⇒ No. of trees $= |A_1 \cdot A_1^T|$

$A_1$ - Reduced Incidence Matrix.

$A_1^T \rightarrow$ The transpose of Reduced Incidence Matrix.

⇒ Reduced Incidence Matrix can also be written as

$$A_1 = [A_T \; A_L]$$

$A_T$ ⇒ coloumns of twings.

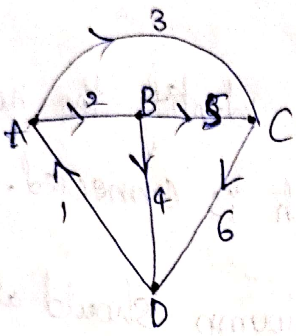
$A_L$ ⇒ Coloumns of links.

⇒ The node corresponding to deleted nod row is (a), as Reference node / Datum node.

*⇒ For a given graph by applying KCL, the reduced incidence matrix can be calculated.

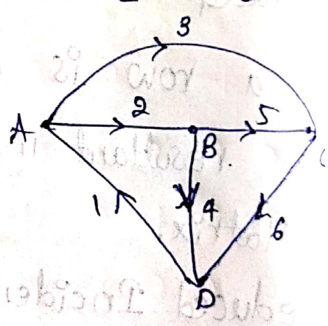
29/09/... Friday

fig :- graph.



Q:- Let us consider the following graph to Calculated the Reduced Incidence matrix.

⇒ Let us consider node 'D' as a reference node. ('D' needs to be deleted).



⇒ Apply KCL to nodes A, B, C.

Case :- 1

Apply KCL to 'A'

$i_1 = i_2 + i_3$

$i_1 - i_2 - i_3 = 0$ —(1).

Case :- 2

Apply KCL to 'B'.

$i_2 = i_5 + i_4$

$i_2 - i_5 - i_4 = 0$ —(2).

Case :- 3

Apply KCL to node 'C'.

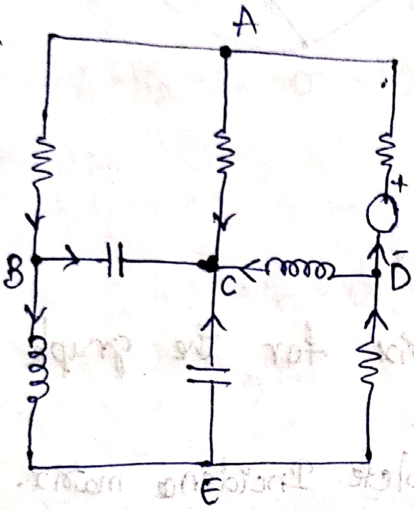
$i_3 + i_5 = i_6$

$i_3 + i_5 - i_6 = 0$ —(3).

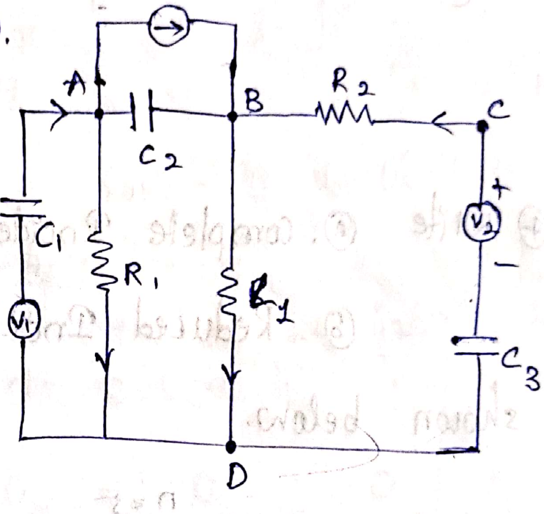
$$A_1 = \begin{array}{c} \text{Nodes} \\ A \\ B \\ C \end{array}\begin{array}{c}\text{Branches} \\ \begin{bmatrix} i_1 & i_2 & i_3 & i_4 & i_5 & i_6 \\ 1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 1 & -1 \end{bmatrix}\end{array}$$

(2). Draw the directed graph for the given networks.

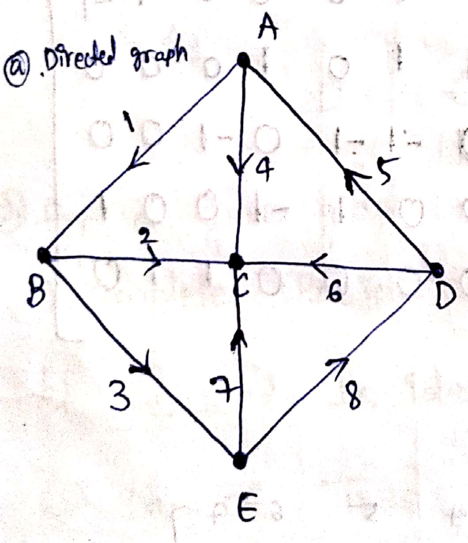
(a).




(b).



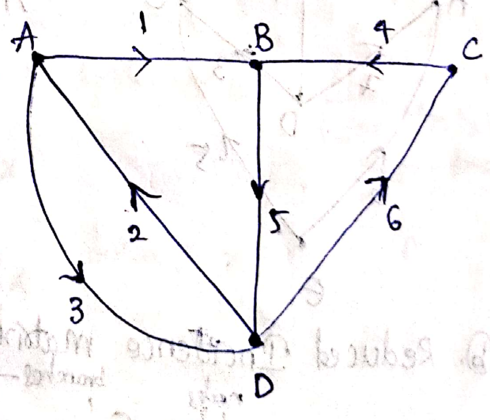

(a) Directed graph




$n = 5$

$b = 8$.

(b). directed graph.

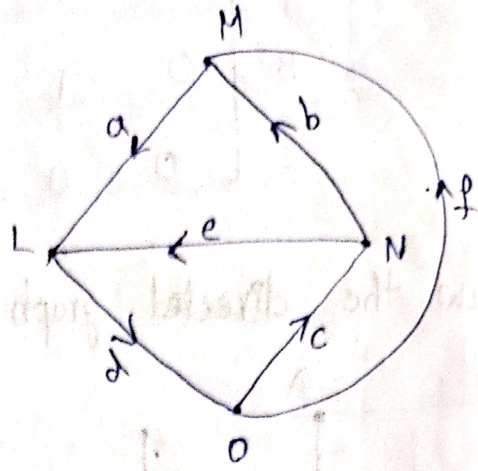



No. of nodes $n = 4$

No. of branches $b = 6$

③. Draw the oriented graph from complete ~~Re~~ Incidence Matrix shown below.

$$
\begin{array}{c|cccccc}
\text{Nodes} \backslash \text{branches} & a & b & c & d & e & f \\
\hline
L & -1 & 0 & 0 & 1 & -1 & 0 \\
M & 1 & -1 & 0 & 0 & 0 & -1 \\
N & 0 & 1 & -1 & 0 & 1 & 0 \\
O & 0 & 0 & 1 & -1 & 0 & 1
\end{array}
$$



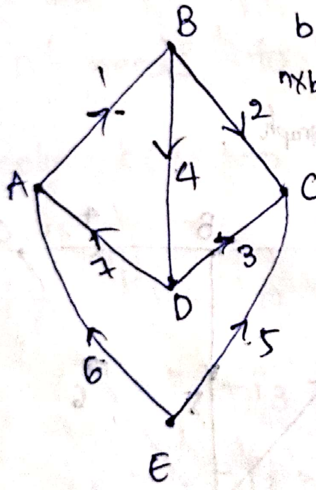

④ write ⓐ. complete Incidence matrix,

ⓑ. Reduced Incidence matrix for the graph shown below.

$n=5$

$b=7$

$n \times b = 5 \times 7$




ⓐ. Complete Incidence matrix.

$$
A = \begin{array}{c|ccccccc}
\text{nodes} \downarrow \backslash \text{branches} \rightarrow & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\
\hline
A & 1 & 0 & 0 & 0 & 0 & -1 & -1 \\
B & -1 & 1 & 0 & 1 & 0 & 0 & 0 \\
C & 0 & -1 & -1 & 0 & -1 & 0 & 0 \\
D & 0 & 0 & 1 & -1 & 0 & 0 & 1 \\
E & 0 & 0 & 0 & 0 & 1 & 1 & 0
\end{array}
$$

ⓑ. Reduced Incidence matrix.

$$
A_1 = \begin{array}{c|ccccccc}
\text{nodes} \downarrow \backslash \text{branches} \rightarrow & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\
\hline
A & 1 & 0 & 0 & 0 & 0 & -1 & -1 \\
B & -1 & 1 & 0 & 1 & 0 & 0 & 0 \\
C & 0 & -1 & -1 & 0 & -1 & 0 & 0 \\
D & 0 & 0 & 1 & -1 & 0 & 0 & 1
\end{array}
$$

Case-① Apply KCL to node 'A'.

$i_6 + i_7 = i_1$

$i_6 + i_7 - i_1 = 0$ —①

Case (2):-

Apply KCL to node 'B'.

$i_1 = i_2 + i_4$

$i_1 - i_2 - i_4 = 0$ —②

Case (3):-

Apply KCL to node 'C'.

$i_2 + i_3 + i_5 = 0$ —③.

Case (4):- Apply KCL to node 'D'.

$i_4 = i_3 + i_7$.

$i_4 - i_3 - i_7 = 0$ — ④.

branches →

$$A_1 = \begin{array}{c} \text{nodes} \downarrow \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{ccccccc} i_1 & i_2 & i_3 & i_4 & i_5 & i_6 & i_7 \end{array} \\ \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & -1 & 0 & -1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & -1 \end{bmatrix} \end{array}_{4\times 7}.$$

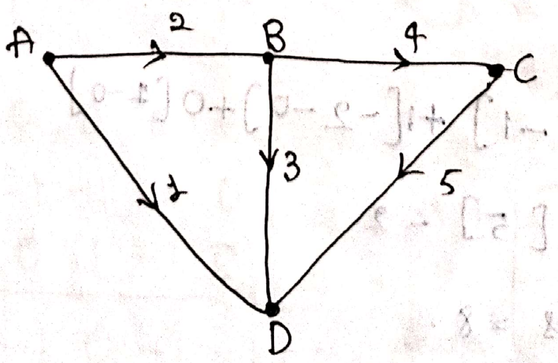
② For a given graph write

a) Complete Incidence matrix.

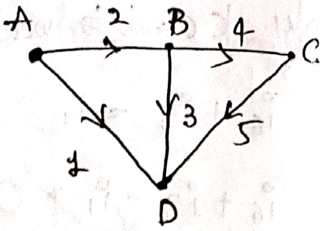
b) Reduced Incidence matrix.

c) No. of Trees.

D:-04/10/2023
Day: wednesday



a) Complete Incidence Matrix.




no. of nodes 'n' = 4 (A, B, C, D)

no. of branches 'b' = 5. (1, 2, 3, 4, 5) → branches

towards (−1)

away (+1)

$$A = \begin{array}{c|c} \text{nodes} \downarrow & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \\ -1 & 0 & -1 & 0 & -1 \end{bmatrix}_{4\times5} \end{array}$$

b) Reduced Incidence Matrix.

$$A_1 = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

c) No. of Trees $= |A_1 \cdot A_1^T|$

$$A_1 \cdot A_1^T = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & 0-1 & 0 \\ 0-1 & 1+1+1 & -1-1 \\ 0 & 0-1 & 1+1 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

$$|A_1 \cdot A_1^T| = 2[6-1] + 1[-2-0] + 0[1-0]$$

$$= 2[5] - 2$$

no. of trees = 8.

(2). The Reduced Incidence matrix is given, calculate.

a) Calculate the no. of trees.

b) complete incidence.

c) Draw the graph.

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ -1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

c) No. of trees.

$$A_1 \cdot A_1^T = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ -1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 0 & -1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \cancel{1+1} & \cancel{-1} & 0 \\ 1+1+1 & \cancel{-1} & -1 \end{bmatrix} = \begin{bmatrix} 1+1 & -1 & 0 \\ -1 & 1+1+1 & -1 \\ 0 & -1 & 1+1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

$$|A_1 \cdot A_1^T| = 2[6-1] + 1[-2-0] + 0[1-0] = 2[5]-2 = 8.$$

no. of trees = 8.

b) Complete Incidence Matrix.

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ -1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

$$\begin{array}{lccccc} A_1 = & 1 & 0 & 0 & 0 & -1 \\ & -1 & -1 & -1 & 0 & 0 \\ + & 0 & 0 & 1 & -1 & 0 \\ \hline \text{sum of elements of columns} & 0 & -1 & 0 & -1 & -1 \\ \hline \text{elements to be added to make} & 0 & 1 & 0 & 1 & 1 \end{array}$$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ -1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

Added row

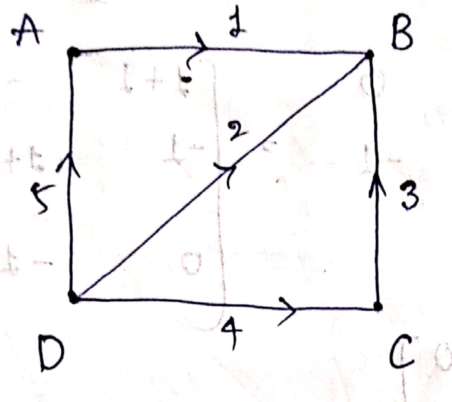
c) Draw the graph.

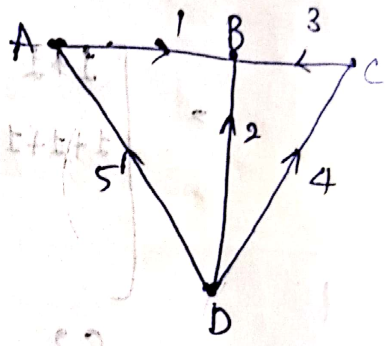
$$A = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ -1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

from the matrix 'A'.

no. of nodes 'n' = 4 (A, B, C, D)

no. of branches 'b' = 5 (1, 2, 3, 4, 5).




③. obtain complete Incidence matrix from Reduced Incidence matrix and draw the graph.

$$\begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & -1 & 1 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & -1 & 1 \end{bmatrix}$$

$$\begin{array}{ccccccc} -1 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}$$

$$\begin{array}{ccccccc} 1 & 0 & 0 & 0 & 0 & 0 & -1 \end{array}$$

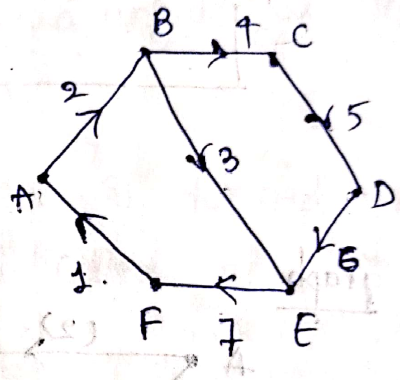
Complete Incidence Matrix.

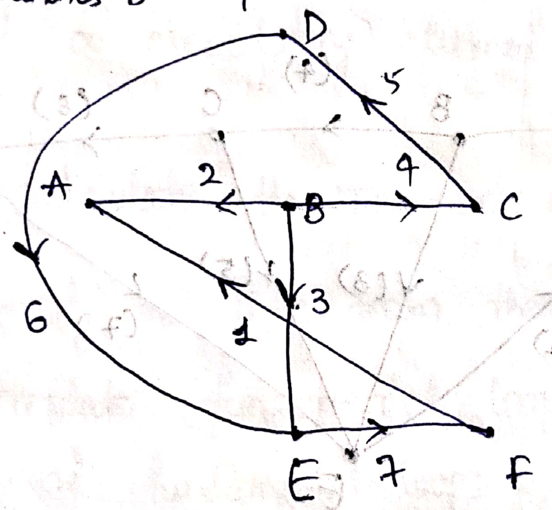
$$A = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \end{array} \begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \end{array}$$

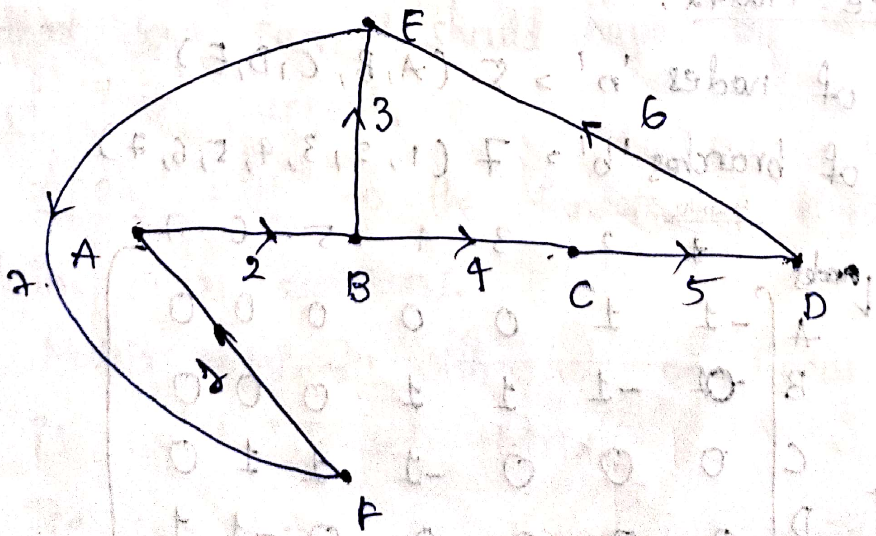
⇒ graph.

no. of nodes 'n' = 6

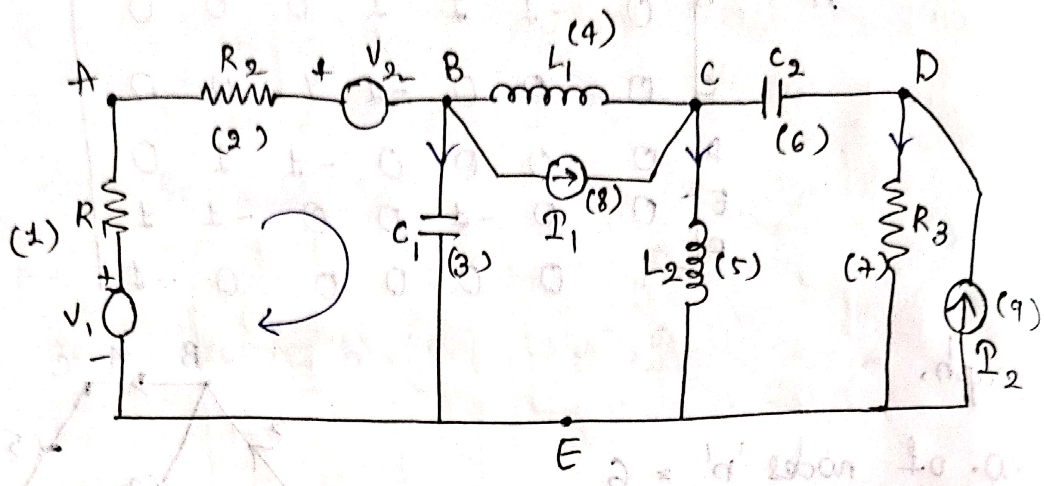
no. of branches 'b' = 7



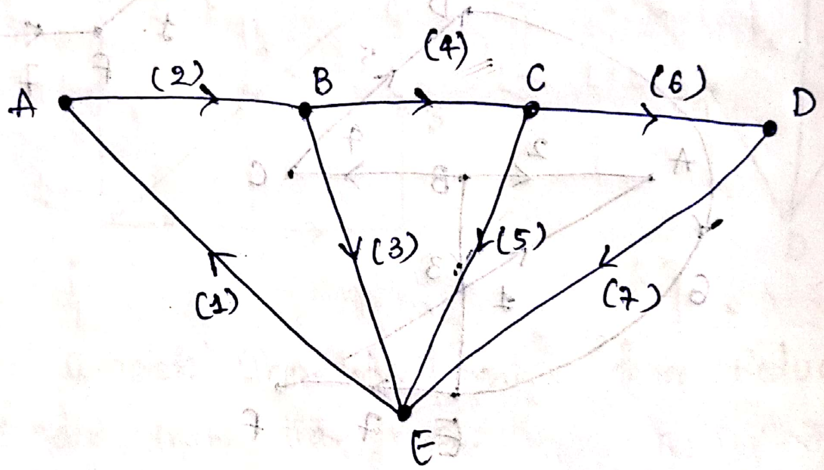



④ Draw the oriented graph and write Incidence matrix for the n/w shown.




graph




Incidence Matrix.

towards (-
away (+

no. of nodes 'n' = 5 (A, B, C, D, E)

no. of branches 'b' = 7 (1, 2, 3, 4, 5, 6, 7).

branches →, nodes ↓

$$A = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ 1 & 0 & -1 & 0 & -1 & 0 & -1 \end{bmatrix} \end{array}$$

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# Tie - set Matrix

⇒ Tie-set Matrix is also called as fundamental circuit Matrix, And fundamental loop (or) f- loop.

Defⁿ :- For a given tree of a graph additional of link will create a closed path called as fundamental loop (or) f-loop (or) Tie-set.

⇒ Each and every Tie-set will have a unique path in the tree.

⇒ when Tie-set (or) fundamental loop is formed then there exist a circulating current called as link current.

⇒ steps to calculate the Tie-set matrix.

Step-1 :- Identify the Tree from the given graph.

Step-2 :- formulate fundamental loops by adding links.

Step-3 :- No. of fundamental loops = no. of links.

Step-3 :- Assign directions to the loop current in such way that loop current should have the same direction as of link current.

Step-4 :- Apply KVL to the fundamental loops and obtain linear equations.

Step-5 :- Matrix obtained with help of linear equations is called Tie-set Matrix.

⇒ Tie-set Matrix is represented with "B".

⇒ The relation between branch currents and loop currents is given by $\boxed{I_b = B^T \cdot I_L}$

$$\boxed{I_b = B^T I_L}$$

$I_b$ = branch currents

$B^T$ = Transpose of Tie-set matrix

$I_L$ = loop (or) link current.

**Note :-**

⇒ $I_b = B^T \cdot I_L$ ⇒ This equation is also known as link current transformation equation.

⇒ Tie-set matrix can be re-arranged by writing coloumns of links in first half and coloumns of twigs in second half of the matrix.

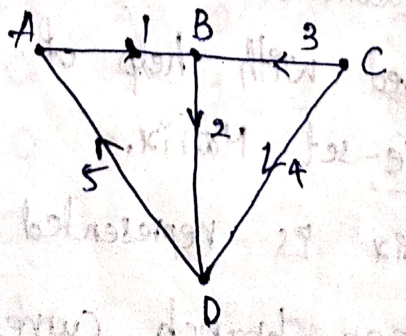
i.e., $B = B_L \,\vdots\, B_T$

$B_L$ = coloumns of links.

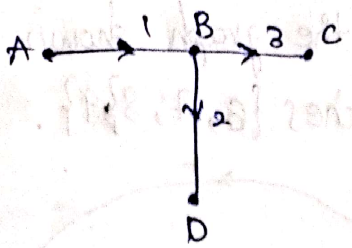
$B_T$ = Coloumns of Twigs.

⇒ It is observed that, $B_L$ is always an unit matrix and hence $B = U \,\vdots\, B_T$

1) Let us consider the following graph to calculate the Tie-set matrix.



step-1 :- Tree



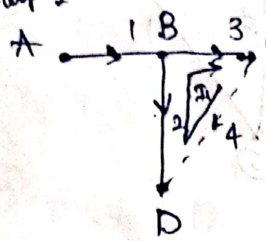

$S_2$ & $S_3$ :- Create f-loops,

no. of f-loops = no. of links

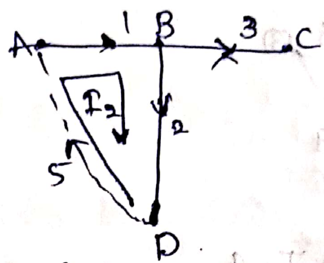
links = 2, {4, 5}.

step-4 :-

f-loop-1




f-loop-2




step-0,

f-loop-1

$V_4 - V_2 + V_3 = 0$

f-loop-2

$V_5 + V_1 + V_2 = 0.$

step-⑤.

$B \cdot V_b = 0$

$$
\begin{array}{ll}
 & \text{branches} \rightarrow \\
\text{f-loops} \downarrow & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\
\begin{array}{l} \text{f loop-1 : } I_1 \\ \text{f loop-2 : } I_2 \end{array} & \begin{bmatrix} 0 & -1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix}
\end{array}
\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix} = 0
$$

Tie-set matrix.

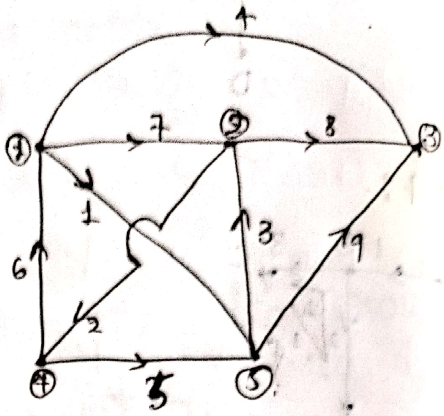
$$B = \begin{bmatrix} 0 & -1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

05/10/2023
Thursday

1) obtain fundamental circuit matrix for the graph shown in fig. choose the tree consisting of branches {6, 7, 8, 9}.

4 Marks

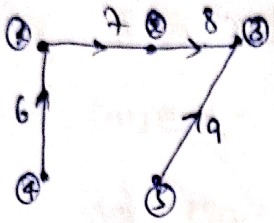
B = ?



no. of nodes $n = 5$

no. of branches $b = 9$

(S1).



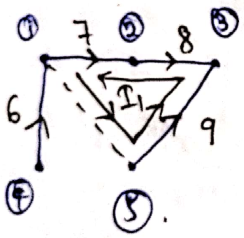
no. of twigs $T = 4$

no. of links $L = 5$ {1, 2, 3, 4, 5}.

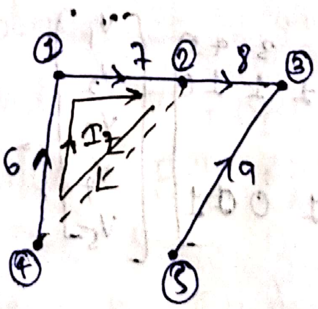
(S2). f-loops.

no. of f-loops = no. of links = 5

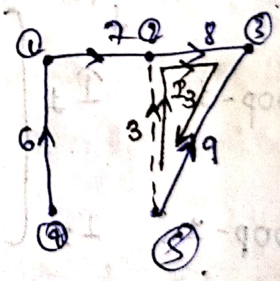
f-loop-1



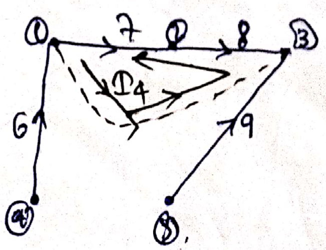
f-loop-2



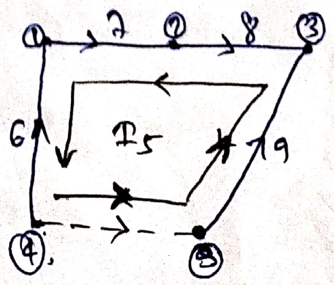
f-loop-3



f-loop-4



f-loop-5



(S4). Apply KVL

f-loop-1

$V_1 + V_9 - V_8 - V_7 = 0$.

f-loop-2:-

$V_2 + V_6 + V_7 = 0$

f-loop-3:-

$V_3 + V_8 - V_9 = 0$

f-loop-4:-

$V_4 - V_8 - V_7 = 0$

f-loop-5:-

$V_5 + V_9 - V_8 - V_7 - V_6 = 0$.

(S5). $B \cdot V_b = 0$.

f-loops ↓ branches →

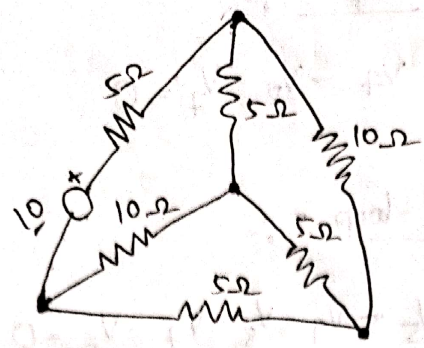
$$\begin{array}{l} \\ \text{f-loop 1 :- } I_1 \\ \text{f-loop 2 :- } I_2 \\ \text{f-loop 3 :- } I_3 \\ \text{f-loop 4 :- } I_4 \\ \text{f-loop 5 :- } I_5 \end{array}
\begin{array}{c} \begin{array}{ccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & -1 & -1 & 1 \end{bmatrix} \end{array}
\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ V_9 \end{bmatrix} = 0$$

(S6).

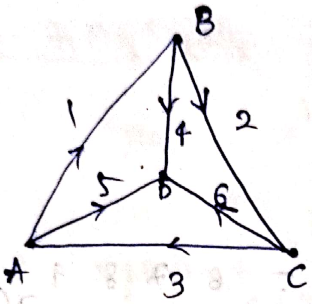
Tie-set Matrix

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & -1 & -1 & 1 \end{bmatrix}$$

(2) For the network shown in figure obtain incidence matrix and determine the relation between loop and branch currents.
9 Marks.




let
no. of nodes $n = 4$ (A, B, C, D)




⇒ from graph.
no. of branches 'b' = 6.

① Calculation of Incidence Matrix A.

$$A = \begin{array}{c} \\ \text{nodes} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \text{branches} \rightarrow \\ \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1 & 0 & -1 & 0 & 1 & 0 \\ -1 & 1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & -1 & -1 \end{bmatrix} \end{array}$$

$n = 4$
$b = 6$

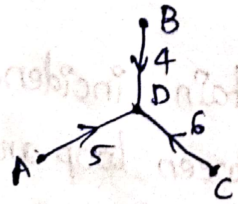
06/10/202
Friday

②. Relation between loop currents & branch currents.

$$I_b = B^T \cdot I_L \quad \text{—} ①$$

Tie-set Matrix :- B.

(S1).

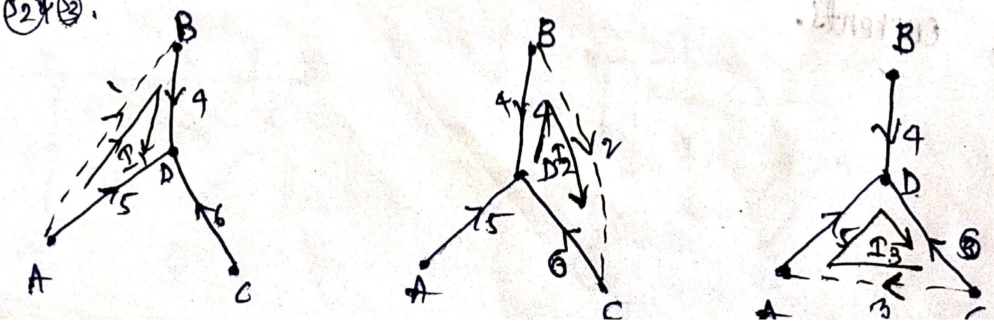



no. of twigs $T = 3$

no. of links $L = 3$ {1, 2, 3}.

we know that no. of f-loops = no. of links = 3.

(S2) & (S3).



s4) Apply KVL.

f-loop-1:- $V_1 + V_4 - V_5 = 0$

f-loop-2:- $V_2 + V_6 - V_4 = 0$

f-loop-3:- $V_3 + V_5 - V_6 = 0$.

5). $B \cdot V_b = 0$.

$$
\begin{array}{l}
\text{f-loops} \downarrow \quad \text{branches} \rightarrow \\
\begin{array}{l} \text{f-loop-1}: I_1 \\ \text{f-loop-2}: I_2 \\ \text{f-loop-3}: I_3 \end{array}
\begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & -1 \end{bmatrix} \end{array}
\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} = 0.
\end{array}
$$

Tie-set matrix 'B'

$$
B = \begin{bmatrix} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & -1 \end{bmatrix}
$$

from eqⁿ ①.

$$I_b = B^T I_L$$

$$
\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}
$$

$i_1 = I_1$

$i_2 = I_2$

$i_3 = I_3$

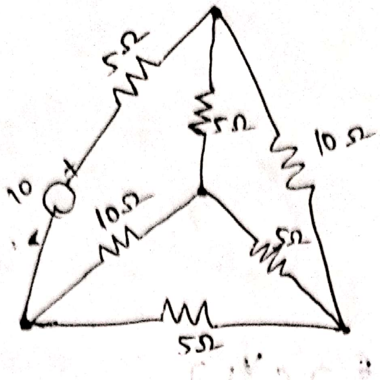
$i_4 = I_1 - I_2$

$i_5 = -I_1 + I_3$

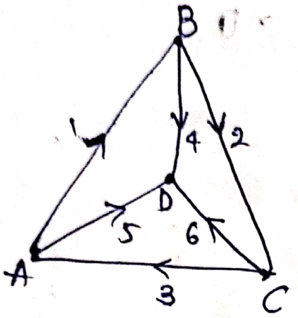
$i_6 = I_2 - I_3$ //.

①. For the network shown in figure. write a Tie-set schedule and then find branch currents and voltages.

15 Marks (or) 10




from the n/w the graph.

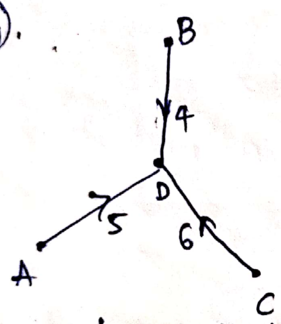



no. of nodes 'n' = 4 (A, B, C, D)

no. of branches 'b' = 6 (1, 2, 3, 4, 5, 6)

Tie-set Matrix 'B'.

(S₁).

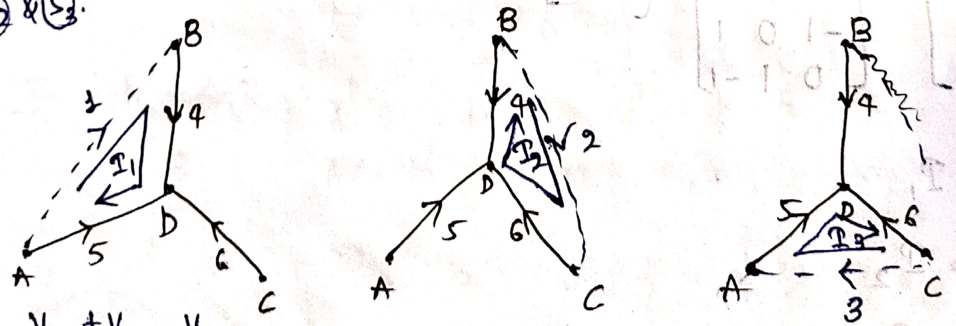



no. of twigs 'T' = 3

no. of links 'L' = 3 {1, 2, 3}

we know that no. of f-loops = no. of links = 3.

(S₂) & (S₃).




$V_1 + V_4 - V_5 = 0$ $\quad$ $V_2 + V_6 - V_4 = 0$ $\quad$ $V_3 + V_5 - V_6 = 0$.

Tie-set schedule

| floops | branches | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 |
| f-loop-1 | 1 | 0 | 0 | 1 | -1 | 0 |
| f-loop-2 | 0 | 1 | 0 | -1 | 0 | 1 |
| f-loop-3 | 0 | 0 | 1 | 0 | 1 | -1 |

Tie-set matrix 'B'

$$B = \begin{bmatrix} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & -1 \end{bmatrix}$$

we know that

$$I_b = B^T . I_L$$

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

branch currents

$i_1 = I_1$

$i_2 = I_2$

$i_3 = I_3$

$i_4 = I_1 - I_2$

$i_5 = I_1 + I_3$

$i_6 = I_2 - I_3$

## ⇒ Cut - set Matrix

Cut - set Defination :- The set of branches whose removal can cut a graph into exactly two parts, is said to be cut-set.

fundamental cut-set.

A cut-set which contains only one twing and can contain any no. of links is said to be fundamental cut-set.

cut-set = 1 - twig and 'n' no. of links.

procedure to calculate cut-set Matrix.

step-1 :- Identify tree and co-tree at a time.

step-2 :- Calculate no. of cut-sets where no. of cut sets

no. of cut sets = no. of twings = (n-1)

step-3 :- Draw the cut-set in such a way that, it should contain only one twing and any no. of links.

step-4 :- Identify the direction of cut-set in such a way that it should be in the same direction of twing.

step-5 :- Apply KCL and obtain linear equations.

step-6 :- Arrange linear equation in form of

$$\boxed{Q . I_b = 0.}$$

where $Q$ = cut-set matrix

$I_b$ = branch currents.

⇒ The relation between branch voltages and cut-set voltages (or) twig voltages is given by

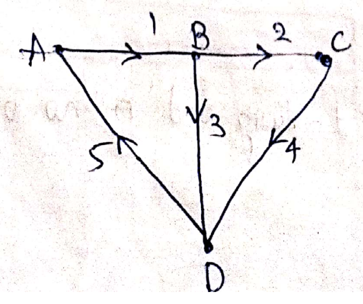
$$\boxed{V_b = Q^T V_t}$$

$V_b$ = branch voltages.

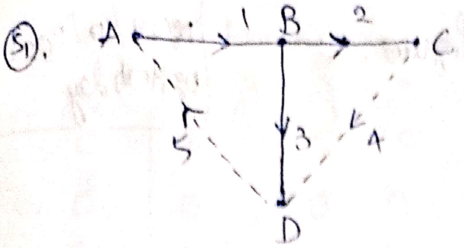
$Q^T$ = Transpose of cut-set matrix.

$V_t$ = twig voltages (or) cut-set voltages.

① Let us consider the following graph to calculate cut-set matrix.

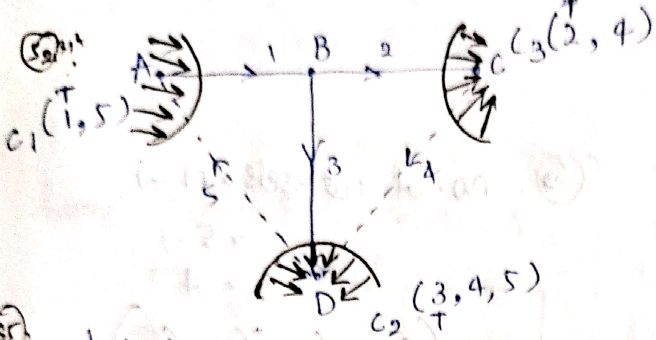


(S1).




no. of cut-sets = no. of twigs = 3.

(S2)




(S5)

⇒ Apply KCL.

$C_1 \Rightarrow i_1 - i_5 = 0$

$C_2 \Rightarrow i_3 + i_4 - i_5 = 0$

$C_3 \Rightarrow i_2 - i_4 = 0.$

⇒ we know that $Q\, i_b = 0$.

(S6). cut-sets ↓ branches →

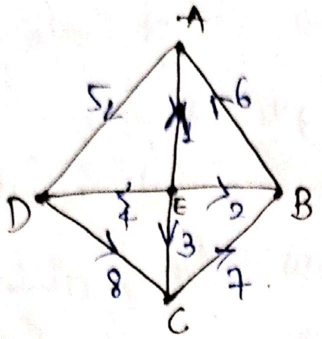
$$\begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 1 & -1 \\ 0 & 1 & 0 & -1 & 0 \end{bmatrix} \end{array} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = 0$$

cut-set matrix.

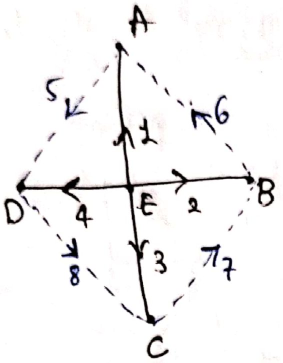
$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 1 & -1 \\ 0 & 1 & 0 & -1 & 0 \end{bmatrix}$$

Dt: 09/10/202
Dt: Holi day

(1) for the directed graph shown in figure. find cut-set matrix.




$(S_1)$.



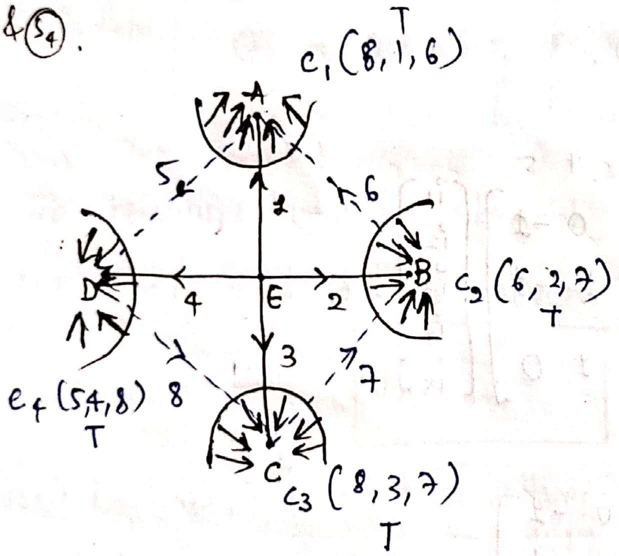

$(S_2)$. no. of cut-sets $= n-1$

$= 5-1$

$= 4$

$(C_1, C_2, C_3, C_4)$.

$(S_3)$ & $(S_4)$.




cut-sets

$$\begin{array}{c} C_1 \\ C_2 \\ C_3 \\ C_4 \end{array} \begin{bmatrix} \\ \\ \\ \\ \end{bmatrix}$$

$(S_5)$. Apply KCL.

$C_1 \Rightarrow i_1 - i_5 + i_6 = 0$

$C_2 \Rightarrow i_2 - i_6 + i_7 = 0$

$C_3 \Rightarrow i_3 - i_7 + i_8 = 0$

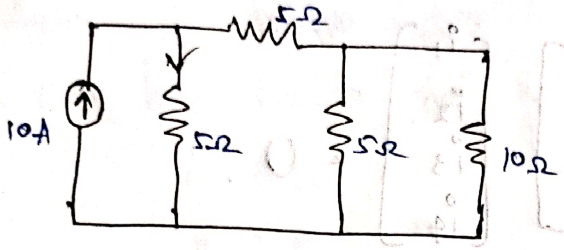
$C_4 \Rightarrow i_4 + i_5 - i_8 = 0$.

$(S_6)$. $Q \cdot I_b = 0$.

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & -1 \end{bmatrix}$$

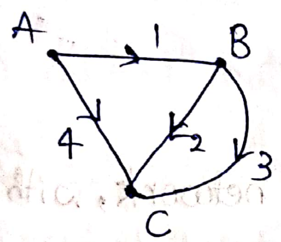
$$\begin{array}{c|c} & \text{branches} \rightarrow \\ \text{cut-sets} \downarrow & \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\ \begin{array}{c} C_1 \\ C_2 \\ C_3 \\ C_4 \end{array} & \begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \\ i_7 \\ i_8 \end{bmatrix} = 0. \end{array}$$

→ For the n/w shown in figure. Draw oriented graph and frame cut-set matrix.

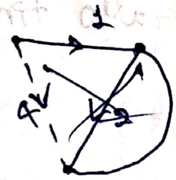


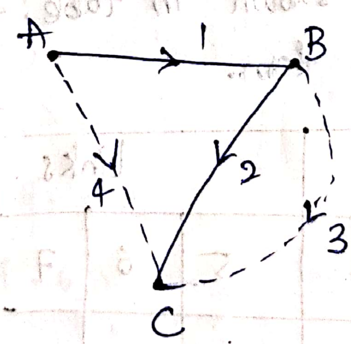

Sol:- Let, the no. of nodes 'n' = 3

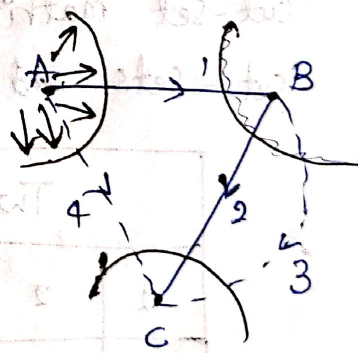
b = 4.



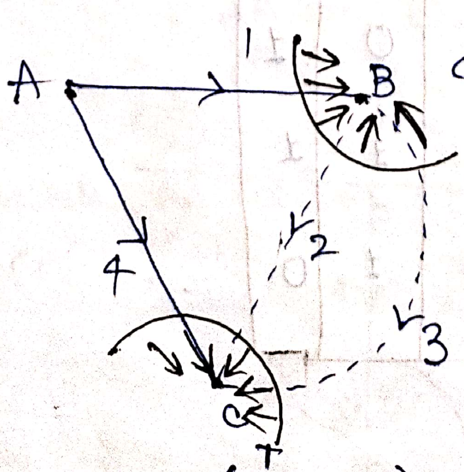

oriented graph.




(S₁).




$C_1(1,2,3)$

$C_2(4,2,3)$

(S5). Apply KCL

$C_1 \Rightarrow i_1 - i_2 - i_3 = 0$

$C_2 \Rightarrow i_4 + i_2 + i_3 = 0.$

(S6). $Q \cdot i_b = 0$.

cut sets ↓ branches →

$$\begin{array}{c} \\ C_1 \\ C_2 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & -1 & -1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \end{array} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = 0.$$

$$Q = \begin{bmatrix} 1 & -1 & -1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

(3). Draw the oriented graph of a network, with fundament cut-set matrix, as shown in table. And also find no. o cut-sets and draw them.

| Twigs | | | | Links | | |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 1 | 0 | 0 | 0 | -1 | 0 | 0 |
| 0 | 1 | 0 | 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 0 | 0 | 1 | 1 |
| 0 | 0 | 0 | 1 | 0 | 1 | 0 |

Sol :- we know that

$$Q = Q_T \,\vdots\, Q_L$$

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

from the above matrix.

⇒ it can be known that no. of cut-sets = ~~no~~ 4 (no. of rows)

$n-1 = 4$

$n = 5$.

⇒ The no. of nodes require to draw oriented graph from the given cut-set matrix are $n = 5$ (A, B, C, D, E).

①. no. of cut-sets = no. of rows of 'Q' 11/10/202

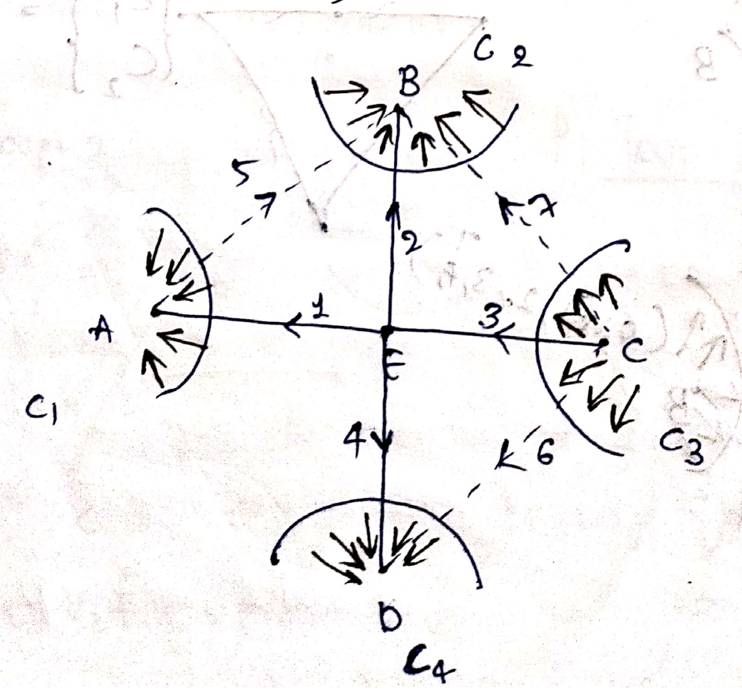
no. of cut-sets = 4.

$C_1 = (\overset{T}{1}, 5)$

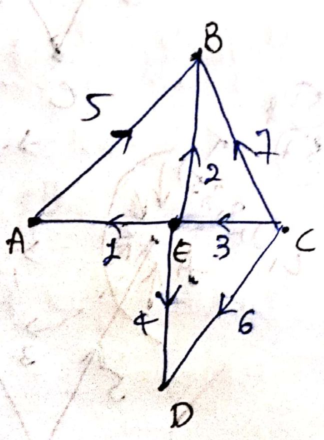
$C_2 = (\overset{T}{2}, 5, 7)$

$C_3 = (\overset{T}{3}, 6, 7)$

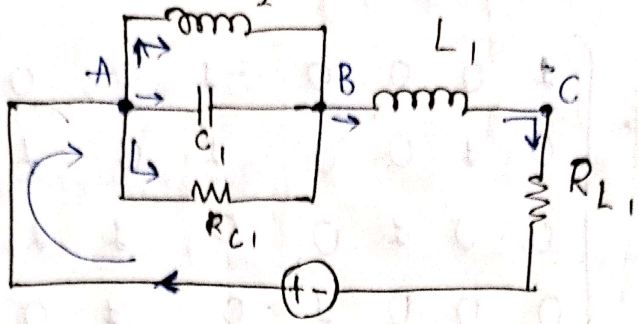
$C_4 = (\overset{T}{4}, 6)$.




oriented graph

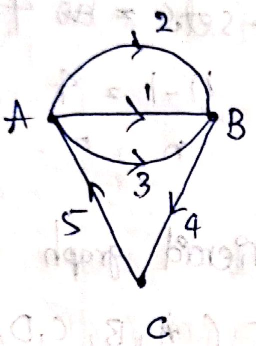


1. Draw the directed graph, tree, cut-set matrix and
10M. Tie-set matrix for the network shown in figure.



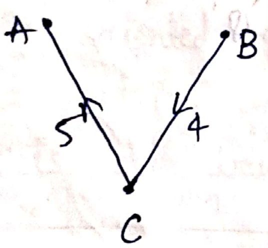

Sol: Graph:

let, the no. of nodes $n = 3$ (A, B, C).




no. of branches $b = 5$.

Tree :-

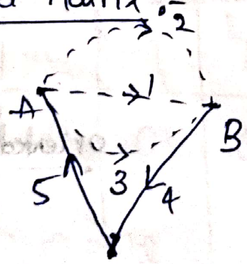



no. of twigs $T = 2$ (4, 5)

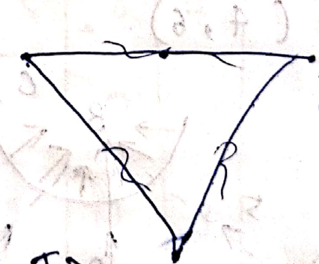
no. of links $L = 3$ (1, 2, 3)

cut-set Matrix :-

(S1).

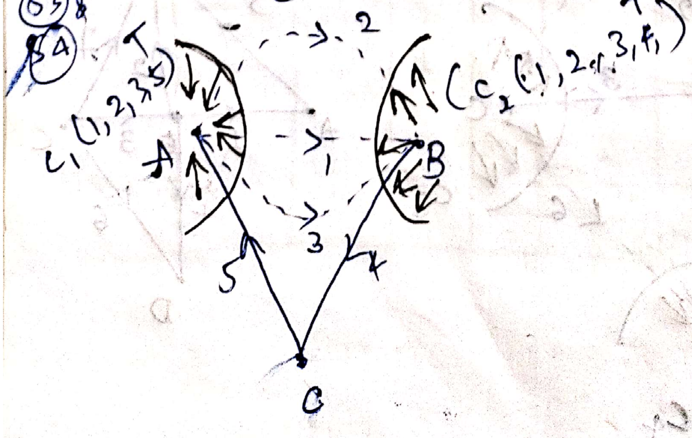



(S2) no. of cutsets = no. of twigs



$\left\{ \begin{matrix} C_1 \\ C_2 \end{matrix} \right\} = 2.$

(S3) & (S4)



(S3). Apply KCL

$C_1 \Rightarrow i_5 - i_3 - i_1 - i_2 = 0$

$C_2 \Rightarrow i_4 - i_3 - i_1 - i_2 = 0$.

$Q \cdot i_b = 0$.

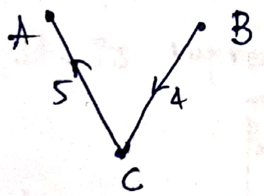
branches →

$$\begin{array}{c|c} \text{cut sets} \downarrow & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} C_1 \\ C_2 \end{array} & \begin{bmatrix} -1 & -1 & -1 & 0 & 1 \\ -1 & -1 & -1 & 1 & 0 \end{bmatrix} \end{array} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = 0$$

$$Q = \begin{bmatrix} -1 & -1 & -1 & 0 & 1 \\ -1 & -1 & -1 & 1 & 0 \end{bmatrix}$$

Tie-set matrix.

(S1). Tree :-

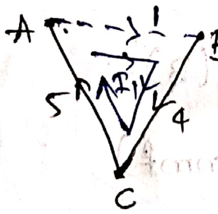



no. of twigs $T = 2(4,5)$

no. of links $L = 3(1,2,3)$.

(S2). no. of f-loops = no. of links $= 3(1,2,3)$

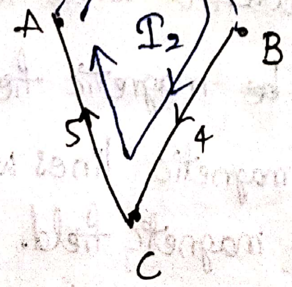
f-loop-1.




Apply KVL

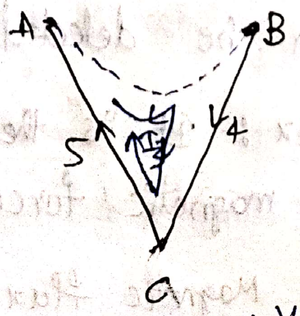
$V_1 + V_4 + V_5 = 0$

f-loop-2




$V_2 + V_4 + V_5 = 0$

f-loop-3




$V_3 + V_4 + V_5 = 0$.

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$B \cdot V_b = 0.$

$$\begin{array}{c|c} & \text{branches} \rightarrow \\ \text{f-loops} \downarrow & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} \text{f-loop-1}: \\ \text{f-loop-2}: \\ \text{f-loop-3}: \end{array} & \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} \end{array} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix} = 0,$$

$$B = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

# Magnetic circuits

Basic terminology of magnetic circuits.

- → magnetic field
- → magnetic flux ($\phi$)
- → magnetic flux Density ($B$)
- → Magnetomotive force (mmf)
- → Magnetic flux intensity ($H$)
- → Reluctance ($S$)
- → permeance.

16/10/2023
Monday

## Magnetic field

The area around a magnetic material where magnet fowrce can be detected, is said to be magnetic field.

Magnetic flux :- It is the amount of magnetic lines which represents magnetic force within the magnetic field.

Magnetic flux is represented with '$\phi$'.

units :- $\phi \rightarrow$ weber (wb).

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magnetic flux density (B) :- The amount of magnetic flux passing through cross sectional area is said to be magnetic flux density.

⇒ It is represented with 'B'. and is given by.

$$\boxed{B = \frac{\phi}{A}} = \frac{\phi}{A} = \frac{wb}{m^2} = wb/m^2.$$

units for magnetic flux density = $wb/m^2$.

Magnetomotive force (mmf) :- Magnetomotive force is the number of turns required to produce magnetic flux.

⇒ It is represented with "mmf" and is given by.

$$\boxed{mmf = NI}$$

N = no. of turns.

I = Current flowing through the coil.

units :- A.T (Ampere. Turns)

Magnetic flux intensity (H) :- Magnetic flux intensity is the mmf per unit length.

⇒ It is represented with "H" and is given by.

$$\boxed{H = \frac{mmf}{l}}$$

units :- A.T/m.

Reluctance (S) :- The property of the magnetic material which opposes flow of magnetic flux, is said to be/is called as Reluctance.

⇒ It is represented with 'S' and is given by

$$\boxed{S = \cancel{\frac{\mu}{lA}}}$$

$$\boxed{S = \frac{l}{\mu A}}$$

where $\mu$ = permeability ⇒ $\mu = \mu_0 \mu_r$

$\mu_0 = 4\pi \times 10^{-7}$

$l$ = length of the magnetic material.

A = Area of cross section.

units :-

permeance :- It is the reciprocal of reluctance and is given by.

$$\text{permeance} = \frac{1}{S}.$$

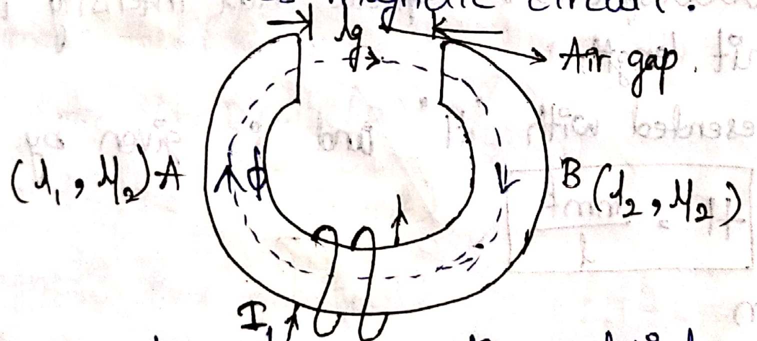
Magnetic circuit :- Magnetic circuit is a closed path which mainly contains magnetic flux ($\phi$).

⇒ Magnetic circuit will also contain different made magnetic materials and different permeabilities.

Magnetic circuits are of 2 types :-

1) series magnetic ckt.
2) parallel magnetic circuit.

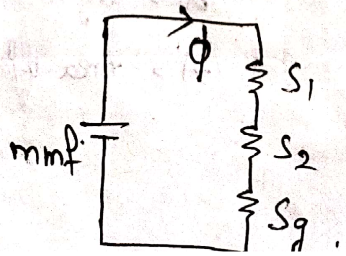
Series Magnetic circuit :- In a magnetic circuit, if there exist only one path for the flow of flux, then it is said to be series magnetic circuit.




let us consider two magnetic materials, which are named as A and B, with two different lengths ($l_1, l_2$) and different permeabilities ($\mu_1, \mu_2$).

⇒ Both the materials are joined with the help of area gap with a length of ($l_g$).

⇒ The equivalent electronic circuit is

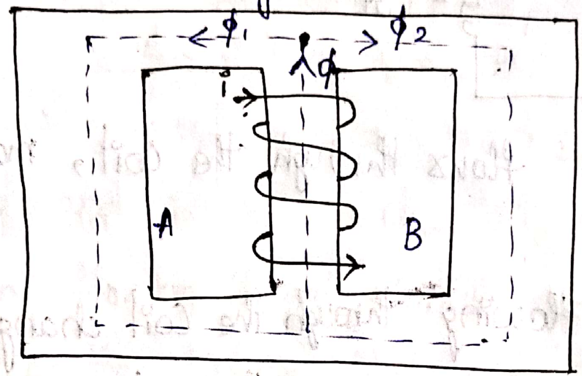


$$\phi = \frac{mmf}{S} \quad -(1)$$

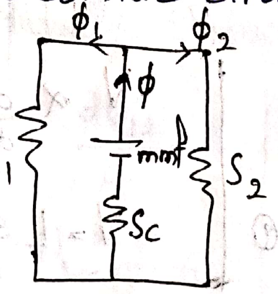
$$S_{total} = S_1 + S_2 + S_g$$

$$\boxed{\phi = \frac{mmf}{S_1 + S_2 + S_g}}$$

**Parallel Magnetic circuit** :- In a magnetic circuit, if there exists more than one path for the flow of flux, then it is said to be parallel magnetic circuit.




let us consider two magnetic material A & B, which are joined parallely with the help of a magnetic coil.
⇒ The equivalent electronic circuit is.




$$\boxed{\phi = \frac{mmf}{S}}$$

$$\phi = \phi_1 + \phi_2$$

$$S_{total} = (S_1 \| S_2) + S_c$$

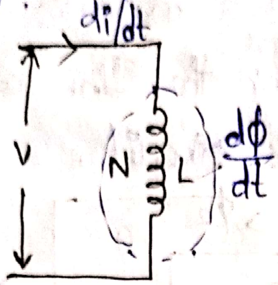
$$\boxed{\phi_{total} = \phi_1 + \phi_2 = \frac{mmf}{(S_1 \| S_2) + S_c}}$$

# Self Inductance (L)

When rate of change of current flows through an Inductor (or) coil, if emf (or) voltage is induced within that coil then it is said to be self Inductance.

⇒ self Inductance is represented with "L"

units :- "Henry".




⇒ when current 'i' flows through the coil, magnetic flux is generated.

⇒ when current flowing through the coil changes with respect to time, then flux across the coil will also change w.r.to time.

⇒ when there is change in flux, voltage is induced across the coil.

$$V \propto \frac{d\phi}{dt} \qquad\qquad V \propto \frac{di}{dt}$$

$$V = N\frac{d\phi}{dt} \; -(1) \qquad\qquad V = L\frac{di}{dt} \; -(2)$$

(1) = (2)

$$\frac{N\,d\phi}{dt} = L\frac{di}{dt}$$

$$N\phi = Li$$

$$\boxed{L = \frac{N\phi}{i}}$$

$N\phi \longrightarrow$ is called as flux linkage and is represented with

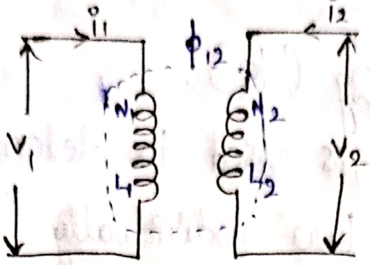
$N\phi \longrightarrow \psi$ (or) $\lambda \Rightarrow \psi = \lambda = N\phi$

## Mutual Inductance ( M )

when rate of ~~curr.~~ change of current flows through one coil, if emf (or) voltage is induced in another coil, then it is said to be Mutual Inductance.

→ It is reperesented with "M".

units :- Henry.




→ when current '$i_1$' flows through coil 1, magnetic flux is generated and it will link the second coil ($\phi_{12}$).

→ when current flowing through coil 1 changes with respect to time, the the linkage flux will also change with respect to time. As a result $V_1$ & $V_2$ are produced.

→ $V_1$ is Called Self induced voltage.

$V_2$ is Called Mutually Induced voltage..

$$V_2 \propto \frac{d\phi_{12}}{dt} \qquad V_2 \propto \frac{di_1}{dt}$$

$$V_2 = N_2 \frac{d\phi_{12}}{dt} \quad \text{--- (1)} \qquad V_2 = M \frac{di_1}{dt} \quad \text{--- (2)}$$

$$(1) = (2)$$

$$N_2 \frac{d\phi_{12}}{dt} = M \frac{di_1}{dt}$$

$$N_2 \phi_{12} = M i_1$$

$$\boxed{M = \frac{N_2 \phi_{12}}{i_1}}$$

$$\boxed{M = \frac{N_1 \phi_{21}}{i_2}}$$

If consider coil 1.

$i_2 \rightarrow \phi_{21}$

$$\frac{di_2}{dt} \rightarrow \frac{d\phi_{21}}{dt} \rightarrow \begin{cases} V_1 \\ V_2 \text{ (self)} \end{cases}$$

$V_1 \propto \frac{d\phi_{21}}{dt}$ | $V_1 \propto \frac{d\phi_2}{dt}$

## Coefficient of Coupling (K):

Coefficient of coupling is used to determine, the amount of coupling between two individually coupled coils.

→ It is represented with 'k'.

Defn :- Coefficient of coupling is the fraction of total flux to the linkage flux, in a network i.e.,

$$K = \frac{\phi_{12}}{\phi_1} \text{ (or) } K = \frac{\phi_{21}}{\phi_2}$$

we know that.

$$M = \frac{N_2 \phi_{12}}{i_1} \quad \text{--- (1)}$$

$$M = \frac{N_1 \phi_{21}}{i_2} \quad \text{--- (2)}$$

(1) X (2).

$$M^2 = \left[\frac{N_2 \phi_{12}}{i_1}\right]\left[\frac{N_1 \phi_{21}}{i_2}\right] \quad \text{--- (3)}$$

multiply & divide eqn (3) with $\phi_1, \phi_2$.

$$M^2 = \left[\frac{N_2 \phi_{12}}{i_1}\right]\left[\frac{N_1 \phi_{21}}{i_2}\right] \frac{\phi_1 \phi_2}{\phi_1 \phi_2}$$

Rearrange the eqn above.

$$M^2 = \left[\frac{N_1 \phi_1}{i_1}\right]\left[\frac{N_2 \phi_2}{i_2}\right]\left[\frac{\phi_{12}}{\phi_1}\right]\left[\frac{\phi_{21}}{\phi_2}\right]$$

$$M^2 = L_1 L_2 k \cdot k$$

$$M^2 = L_1 L_2 k^2$$

$$k^2 = \frac{M^2}{L_1 L_2} \Rightarrow \boxed{k = \frac{M}{\sqrt{L_1 L_2}}}$$

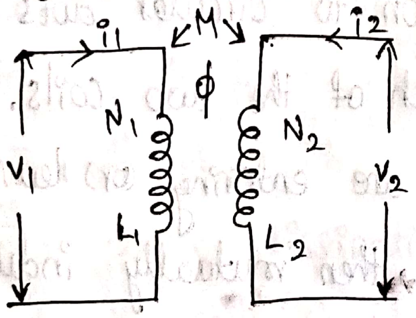
The value of k lies in between 0 and 1 i.e.,

$$0 \leq k \leq 1.$$

⇒ If $k = 1$, it indicates that two coils are strongly coupled. Iron core has $k = 1$.

⇒ If $k = 0$, it indicates that two coils are, coupling between two coils is very poor.

## DOT CONVENTION




$$V_1 = L_1 \frac{di_1}{dt} \pm M \frac{di_2}{dt} \quad \text{—①}$$

$$V_2 = L_2 \frac{di_2}{dt} \pm M \frac{di_1}{dt} \quad \text{—②}$$

⇒ from eqⁿ ① it can be observed that $V_1$ is called as self Induced voltage with respect to $i_1$.

⇒ $V_1$ is also called as Mutually Induced voltage w.r.t $i_2$.

⇒ From eqⁿ ②, it can be observed that $V_2$ is called as ~~Mutua~~ self Induced voltage w.r.t $i_2$. And it is also called as Mutually Induced voltage, w.r.t $i_1$.

⇒ from eqⁿ ① & ②. it is observed that

18/10/20..

1) Self Induced voltages have +ve polarity.

2) Mutually Induced voltages have -ve polarity either +ve (or) -ve polarities.

⇒ The polarity for Mutually Induced voltage can be decided based on the direction of the winding of the coil, but it is complex in analysis.

Usage of dot convention.

⇒ Dot convention is used to decide polarities for Mutual induced voltages.

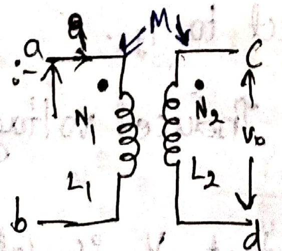
⇒ Dot Rules

Inorder to apply dot convention to circular dots are placed at one end of each of the two coils.

Rule-1 :- If both the currents are entering (or) leaving through the dotted terminals, then mutually induced voltage will have +ve polarity.

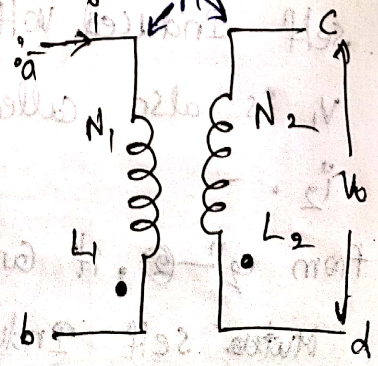
Rule-2 :- If one current is entering through one dotted terminals and if the another current is leaving through the another dotted terminal, then mutually induced voltage will have a -ve polarity.

Ex-1 :-



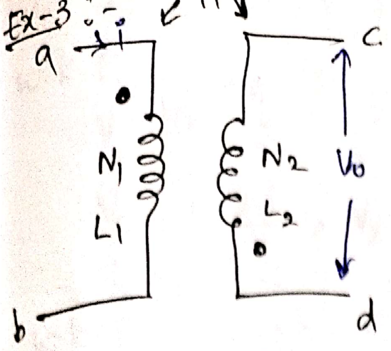

$V_0 = +M\frac{di}{dt}$

Ex-2 :-



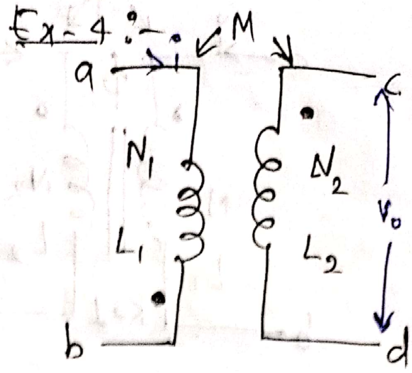

$V_0 = +M\frac{di}{dt}$

Ex-3 :-




$$V_0 = -M\frac{di}{dt}$$

Ex-4 :-




$$V_0 = -M\frac{di}{dt}$$

# Ideal Transformer

Transformer is a static device, with two (or) more winding on a same magnetic material.

⇒ The winding (or) coil which is connected to the source is called primary coil. And the coil which is connected to any load is called as secondary coil.

⇒ A Transformer with.

Conditions

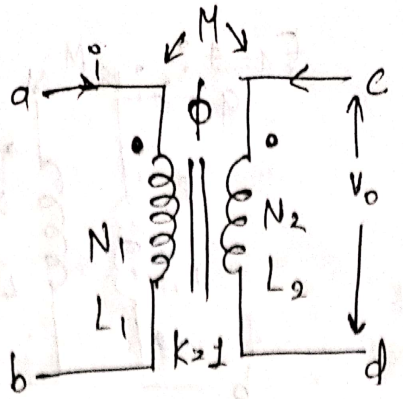
1) zero power dissipation i.e., the internal resistance of both the coils should be zero. $(R_1 = R_2 = 0)$.

2). The self Inductances of both the coils must be large i.e., $(L_1 = L_2 = \infty)$.

3) Coefficient of coupling must be '1' i.e., $(K = 1)$.

is called Ideal Transformer.

⇒ The Iron core is the only one having the coefficient of coupling $(K) = 1$.




$$V \propto \frac{d\phi}{dt}$$

$$V = N\frac{d\phi}{dt} \quad —①$$

$$V \propto \frac{di}{dt}$$

$$V = L\frac{di}{dt} \quad —②.$$

$eq^n \rightarrow ① = ②.$

$$\frac{N\,d\phi}{dt} = \frac{L\,di}{dt} \Rightarrow N\phi = Li$$

$$\therefore \boxed{L = \frac{N\phi}{i}}$$

$$\left[\because \phi = \frac{mmf}{S}\right].$$

$$L = \frac{N}{i}\left[\frac{mmf}{S}\right]$$

$$[\because mmf = Ni]$$

$$L = \frac{N}{i}\left[\frac{Ni}{S}\right] \Rightarrow \boxed{L = \frac{N^2}{S}}$$

$$L \propto N^2$$

$$\frac{L_2}{L_1} = \frac{N_2^2}{N_1^2}$$

but w.k.T $a = \dfrac{N_2}{N_1}$

$a \rightarrow$ Turns ratio (n).

$$\therefore \frac{L_2}{L_1} = \frac{N_2^2}{N_1^2} = a^2$$

$$\boxed{\begin{aligned} a &= \frac{N_2}{N_1} \\ a &= \sqrt{\frac{L_2}{L_1}} \end{aligned}}$$

case(i) :- 'a' w.r.t V.

w.k.T.

$$V = N\frac{d\phi}{dt}$$

$$V_1 = N_1\frac{d\phi}{dt}, \quad V_2 = N_2\frac{d\phi}{dt}$$

w.k.T

$$a = \frac{N_2}{N_1}$$

$$N_2 = \frac{V_2}{d\phi/dt}, \quad N_1 = \frac{V_2}{d\phi/dt} \Rightarrow a = \frac{V_2}{\cancel{d\phi/dt}} \times \frac{\cancel{d\phi/dt}}{V_1}$$

$$\therefore \boxed{a = \frac{V_2}{V_1}}$$

case(ii) :- 'a' w.r.t 'i'.

$$V = L\frac{di}{dt}$$

$$V_1 = L_1\frac{di_1}{dt}, \quad V_2 = L_2\frac{di_2}{dt}$$

w.k.T, $a^2 = \frac{L_2}{L_1}$

$$L_2 = \frac{V_2}{di_2/dt}, \quad L_1 = \frac{V_1}{di_1/dt}$$

$$a^2 = \frac{V_2}{\cancel{di_2/dt}} \times \frac{\cancel{di_1/dt}}{V_1} = \frac{V_2}{V_1} \cdot \frac{i_1}{i_2} \qquad \left[\because a = \frac{V_2}{V_1}\right]$$

$$a^2 = \frac{V_2}{V_1} \cdot \frac{i_1}{i_2}$$

$$a^2 = a \cdot \frac{i_1}{i_2}$$

$$\therefore \boxed{a = \frac{i_1}{i_2}}$$

Case (iii) :- input & output Impedance.

$$Z_{in} = \frac{V_1}{I_1} , \quad Z_{out} = \frac{V_2}{I_2}$$

$$Z_{in} = \frac{V_2}{a.a.I_2} = \frac{V_2}{a^2 I_2} = \frac{1}{a^2} \cdot \frac{V_2}{I_2} = \frac{1}{a^2} Z_{out}$$

$$\boxed{Z_{in} = \frac{1}{a^2} Z_{out}}$$

19/10/2023

## Concept of Reactance

Def$^n$:- The opposition offered by only the capacitor (or) only the inductor to the flow of A.C signal is called Reactance.

⇒ It is represented with '$X$'.

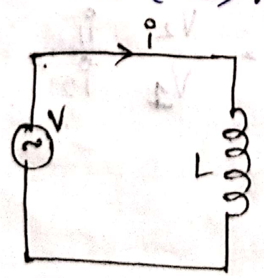
⇒ Units are ohms ($\Omega$).

Reactances are of two types :- 1) Inductive Reactance
2) Capacitive Reactance.

Inductive Reactance :- The opposition offered by inductor to the flow of A.C signal, is known as Inductive reactance.

⇒ It is represented with '$X_L$'.

⇒ units are ohms ($\Omega$).




Let

$$i = I_m \sin(\omega t + \theta) \quad \text{—} (1)$$

polar form eq$^n$ — (1) can be

$$i = I_m \angle \theta \quad \text{—} (2)$$

$$V = L\frac{di}{dt}$$

$$= L\frac{d}{dt}\left[I_m \sin(\omega t+\theta)\right]$$

$$= L \cdot I_m \cdot \frac{d}{dt}\left(\sin(\omega t+\theta)\right)$$

$$= L \cdot I_m \cos(\omega t+\theta)\cdot\omega \qquad \left[\because \frac{d}{dt}\sin^2\theta = \cos 2\theta \cdot 2\right]$$

$$V = \omega_L \cdot I_m \cos(\omega t+\theta)$$

$$= \omega_L I_m \sin(\omega t+\theta+90)$$

$$= \omega_L I_m \angle\theta+90^\circ \qquad \left[\because \angle\theta+90 = \angle\theta\cdot\angle 90\right]$$

$$= \omega_L I_m \angle\theta\cdot\angle 90^\circ \qquad \left[\because \angle 90 = j/i\right]$$

$$= \omega_L I_m \angle\theta\cdot j$$

from $\Rightarrow eq^n$ (2).

$$V = j\omega_L i$$

$$V = jX_L i$$

where $\boxed{X_L = \omega_L}$ is the inductive reactance.

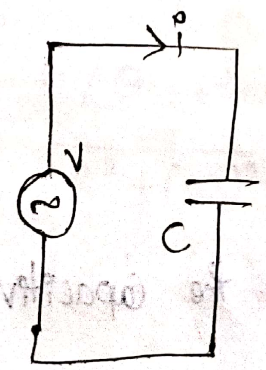
Capacitive Reactance :- The opposition offered by only the capacitor to the flow of A.C signal, is known as Capacitive reactance.

$\sin(-90+\theta) \leftarrow \cos\theta$

$\Rightarrow$ It is represented with '$X_c$'.

$\Rightarrow$ units are ~~for~~ ohms ($\Omega$).

$$\frac{1}{j} = i = -j$$



Let

$$i = I_m \sin(\omega t + \theta) \rightarrow (1)$$

polar form of eq$^n$ (1) can be

$$i = I_m \angle \theta \rightarrow (2)$$

$$V = \frac{1}{c}\int i\,dt$$

$$= \frac{1}{c}\int I_m \sin(\omega t + \theta)dt$$

$$= \frac{I_m}{c}\int \sin(\omega t + \theta)dt \quad [\because \sin(-90+\theta) = -\cos\theta]$$

$$= \frac{I_m}{c}\left[\frac{-\cos(\omega t + \theta)}{\omega}\right]$$

$$= \frac{-I_m}{\omega c}\cos(\omega t + \theta) = \frac{I_m}{\omega c}\sin(\omega t + \theta - 90).$$

$$= \frac{I_m}{\omega c}\angle\theta - 90^\circ = \frac{I_m}{\omega c}\cdot\frac{\angle\theta}{\angle 90^\circ} \quad \left[\because \angle\theta - 90^\circ = \frac{\angle\theta}{90}\right]$$

$$= \frac{I_m}{\omega c}\cdot\frac{\angle\theta}{j} \quad [\because \angle 90 = j/i] \quad [\because j^2 = -1].$$

from $\Rightarrow$ eq$^n$ (2) $\quad e^{j\theta} = \cos\theta + j\sin\theta$

$$= \frac{i}{j\omega c}$$

$$V = i\left(\frac{-j}{\omega c}\right) \quad \left(\because \frac{1}{j} = -j\right)$$

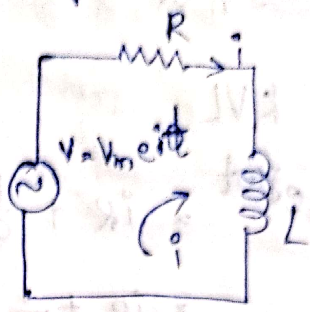
$$V = -i \cdot jX_c$$

where

$$\boxed{X_c = \frac{1}{\omega c}}$$ is the capacitive reactance.

# Concept of Impedance

Def$^n$ :— The opposition offered by $R_L$, $R_C$ (or) $RLC$ network to the flow of a.c signal, is known as Impedance.




→ It is represented with 'Z'.

→ units are ohms ($\Omega$).

Case(i) :- RL network.

Let $V = V_m e^{jt}$

$i = I_m e^{jt}$

Apply KVL

$$V_m e^{jt} = iR + L\frac{di}{dt}$$

$$V_m e^{jt} = I_m e^{jt} R + L\frac{d}{dt}[I_m e^{jt}] \qquad \left[\because \frac{d}{dt} e^{jt} = e^{jt} \cdot j\right]$$

$$= I_m e^{jt} R + L I_m e^{jt} \cdot j$$

$$V_m e^{jt} = I_m e^{jt} [R + jL]$$

$$V_m = I_m [R + jL] \Rightarrow \frac{V_m}{I_m} = R + jL \Rightarrow Z = R + jL$$

If $V = V_m e^{j\omega t}$ & $i = I_m e^{j\omega t}$ then

$$\boxed{Z = R + j\omega L} \rightarrow ① \text{ Impedance of } R_L \text{ network.}$$

polar form of eq$^n$ ①.

$Z = r\angle\theta$ where

$r = \sqrt{R^2 + (\omega L)^2}$

$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$.

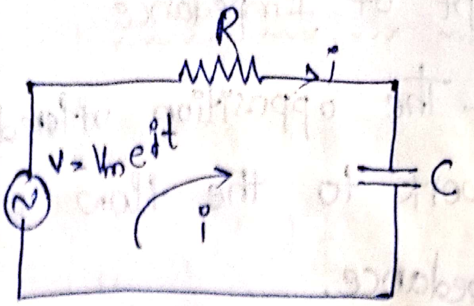
$\because a + jb$

$= r\angle\theta$

$\Rightarrow r = \sqrt{a^2 + b^2}$

$\theta = \tan^{-1}\left(\frac{b}{a}\right)$

Case (ii) :- RC network




let $V = V_m e^{j\omega t}$

$i = I_m e^{j\omega t}$

Apply KVL

$$V_m e^{j\omega t} = iR + \frac{1}{C}\int i\,dt$$

$$= iR + \frac{1}{C}\int I_m e^{j\omega t}\,dt$$

$$= iR + \frac{I_m}{C}\left[\frac{e^{j\omega}}{\omega}\right]$$

[∵ $\int e^{j\omega t} = \frac{e^{j\omega t}}{j\omega}$]

$$= I_m e^{j\omega t} R + \frac{1}{C}\int I_m e^{j\omega t}\,dt$$

$$= I_m e^{j\omega t} R + \frac{I_m}{C}\left[\frac{e^{j\omega t}}{j\omega}\right]$$

$$V_m e^{j\omega t} = I_m e^{j\omega t}\left[R + \frac{1}{j\omega C}\right]$$

$$V_m = I_m\left[R - \frac{j}{\omega C}\right]$$

$$\frac{V_m}{I_m} = R - \frac{j}{\omega C}$$

$\boxed{Z = R - \frac{j}{\omega C}}$ —(1) is Impedance of RC n/w.

polar form of eq$^n$ (1) is

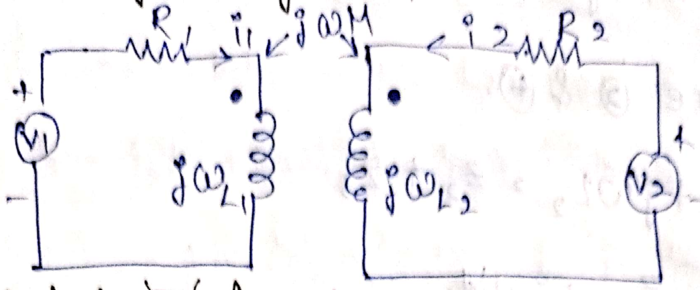
$Z = r\angle\theta$ where

$$r = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

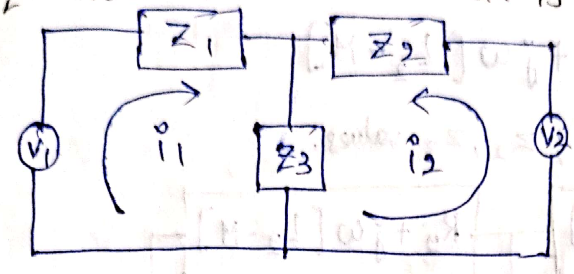
$$\theta = \tan^{-1}\left(\frac{1}{\omega C R}\right)$$

# Equivalent T for magnetically coupled circuits.

case-(i) :- Magnetically coupled circuit.




Equivalent 'T' for above ckt is.




Apply KVL to Magnetic circuit.

$$V_1 = i_1 R_1 + j\omega L_1 i_1 + j\omega M i_2$$

$$V_1 = i_1 [R_1 + j\omega L_1] + j\omega M i_2 \quad — (1).$$

$$V_2 = i_2 R_2 + j\omega L_2 i_2 + j\omega M i_1$$

$$V_2 = i_2 [R_2 + j\omega L_2] + j\omega M i_1 \quad — (2).$$

Apply KVL to 'T' ckt.

$$V_1 = Z_1 i_1 + Z_3 [i_1 + i_2]$$

$$V_1 = i_1 [Z_1 + Z_3] + Z_3 i_2 \quad — (3).$$

$$V_2 = i_2 Z_2 + Z_3 [i_1 + i_2]$$

$$V_2 = i_2 [Z_2 + Z_3] + Z_3 i_1 \quad — (4).$$

Compare eq$^n$ - (1) & (3).

$$R_1 + j\omega L_1 = Z_1 + Z_3$$

$$\boxed{j\omega M = Z_3}$$

$$R_1 + j\omega L_1 = Z_1 + j\omega M$$

$$Z_1 = R_1 + j\omega L_1 - j\omega M$$

$$Z_1 = R_1 + j\omega [L_1 - M]$$

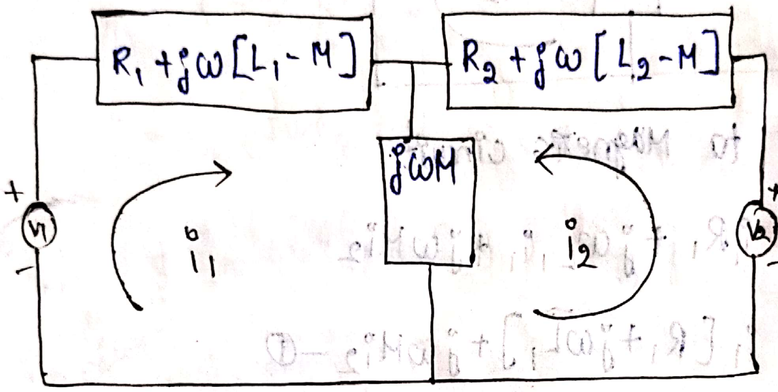
Compare ③ & ④.

$$R_2 + j\omega L_2 = Z_2 + Z_3$$

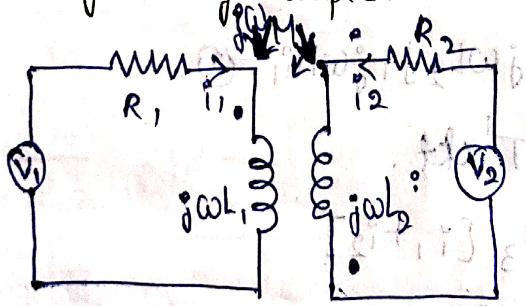
$$Z_3 = j\omega M$$

$$Z_2 = R_2 + j\omega [L_2 - M]$$

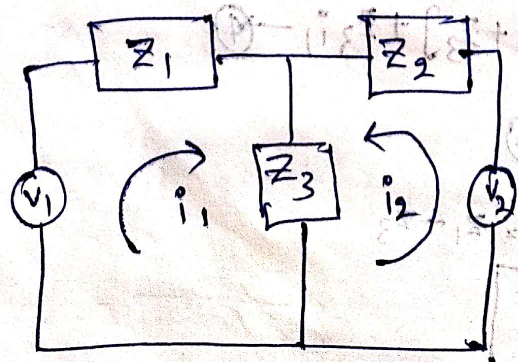
Equivalent 'T' ckt with $Z_1, Z_2, Z_3$ values.




Case (ii):- Magnetically coupled circuit with opposite direction




equivalent 'T' for above ckt:



Apply KVL to Magnetic ckt.

$V_1 = i_1 R_1 + j\omega L_1 i_1 - j\omega M i_2$

$V_1 = i_1 [R_1 + j\omega L_1] - j\omega M i_2$ —— ①

$V_2 = i_2 R_2 + j\omega L_2 i_2 - j\omega M i_1$

$V_2 = i_2 [R_2 + j\omega L_2] + j\omega M i_1$ —— ②.

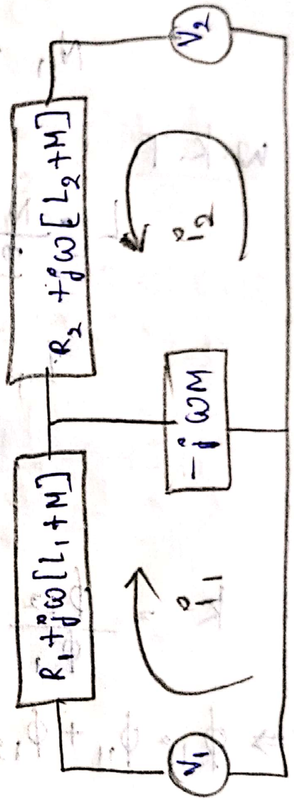
Apply KVL to 'T' circuit.

$V_1 = Z_1 i_1 + Z_3 [i_1 + i_2]$

$V_1 = i_1 [Z_1 + Z_3] + Z_3 i_2$ —— ③.

$V_2 = i_2 Z_2 + Z_3 [i_1 + i_2]$.

$V_2 = i_2 [Z_2 + Z_3] + Z_3 i_1$ —— ④.




Compare ① & ③:

$R_1 + j\omega L_1 = Z_1 + Z_3$

$\boxed{-j\omega M = Z_3}$

$R_1 + j\omega L_1 = Z_1 - j\omega M$

$Z_1 = R_1 + j\omega L_1 + j\omega M$

$\boxed{Z_1 = R_1 + j\omega [L_1 + M]}$

Compare ② & ④.

$R_2 + j\omega L_2 = Z_2 + Z_3$

$\boxed{-j\omega M = Z_3}$

$R_2 + j\omega L_2 = Z_2 - j\omega M$

$Z_2 = R_2 + j\omega L_2 + j\omega M$

$\boxed{Z_2 = R_2 + j\omega [L_2 + M]}$

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# PROBLEMS

1) Coil 1 of a pair of a coupled coils has a continuous current of 5 Amps and corresponding fluxs $\phi_{11}$ & $\phi_{12}$ are 0.2 and 0.4 milli weber. If the turns are $N_1 = 500$, $N_2 = 1500$. Then find $L_1, L_2$ and $k$.

Sol:- Given.

$I = 5A$ | $L_1 = ?$
$\phi_{11} = 0.2$ mwb | $L_2 = ?$
$\phi_{12} = 0.4$ mwb | $M = ?$
$N_1 = 500$, $N_2 = 1500$ | $k = ?$

$m = 10^{-3}$

w.k.T.

$$L = \frac{N\phi}{I} \Rightarrow L_1 = \frac{N_1\phi_{11}}{I}, \quad L_2 = \frac{N_2\phi_{12}}{I}$$

$$L_1 = \frac{500 \times 0.2 \times 10^{-3}}{5}, \quad L_2 = \frac{1500 \times 0.4 \times 10^{-3}}{5}$$

$$= 0.02H \qquad\qquad = 0.12H$$

$$k = \frac{\phi_{12}}{\phi_1}$$

$$\Rightarrow \phi_1 = \phi_{11} + \phi_{12} = 0.6 \text{ mwb}.$$

$$= \frac{0.4 \times 10^{-3}}{0.6 \times 10^{-3}} = 0.67$$

$$k = \frac{M}{\sqrt{L_1 L_2}}$$

$$M = k\sqrt{L_1 L_2}$$

$$= 0.67\sqrt{0.02 \times 0.12}$$

$$= 0.029 H$$

$$= 0.03H$$

(2). The number of turns in two coupled coils is 250 & 750. When 2.5 Amps. current flows in coil 1, the total flux in this coil is 0.3 mwb. And the flux linking the second coil is 0.15 wb. Determine $L_1, L_2, M, k$.

Sol: Given,

$N_1 = 250$ | $L_1 = ?$
$N_2 = 750$ | $L_2 = ?$
$I = 2.5A$ | $M = ?$
$\phi_{11} = 0.3\ mwb$ | $k = ?$
$\phi_{12} = 0.15\ wb$

W.K.T.

$$L = \frac{N\phi}{I} \Rightarrow L_1 = \frac{N_1\phi_{11}}{I}, \quad L_2 = \frac{N_2\phi_{12}}{I}$$

$$= \frac{250 \times 0.3 \times 10^{-3}}{2.5}, \quad L_2 = \frac{750 \times 0.15}{2.5}$$

$$L_1 = 0.03\ H, \quad L_2 = 45H$$

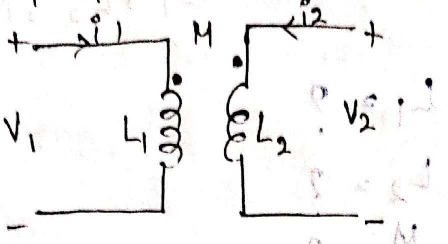
$$k = \frac{\phi_{12}}{\phi_1} = \frac{0.1503}{0.15} = 0.99 \simeq 1.002$$

$$\phi_1 = \phi_{11} + \phi_{12} = 0.3 \times 10^{-3} + 0.15$$
$$= 0.1503$$

$$k = \frac{M}{\sqrt{L_1L_2}}$$

$$M = k\sqrt{L_1L_2}$$
$$= 1\sqrt{0.03 \times 45} = 1.16189.$$
$$= 1.15H$$

③. In the ckt shown in fig. $L_1 = L_2 = 5\mu H$ and $M = 1\mu H$. Compute $V_1$ & $V_2$. If $i_1 = 3\cos 150t$ mA, $i_2 = 4\sin 150t$ mA.




$i_1 = 3\cos 150t$ mA

$i_2 = 4\sin 150t$ mA.

let
Apply KVL

$$V_1 = L_1 \frac{di_1}{dt} + M\frac{di_2}{dt}$$

$$= 5\times10^{-6} \cdot \frac{d}{dt}\left[(3\cos 150t)\times10^{-3}\right] + 10^{-6}\frac{d}{dt}\left[(4\sin 150t)10^{-3}\right]$$

$$= (5\times10^{-6})(3\times10^{-3})\frac{d}{dt}(\cos 150t) + (10^{-6})(4\times10^{-3})\frac{d}{dt}(\sin 150t)$$

$$= 15\times10^{-9}(-\sin 150t)150 + 4\times10^{-9}(\cos 150t)150.$$

$$V_1 = 150\times10^{-9}\left[4\cos 150t - 15\sin 150t\right] V.$$

$$= 150\{4\cos 150t - 15\sin 150t\}\, nV.$$

$$V_2 = L_2\frac{di_2}{dt} + M\frac{di_1}{dt}$$

$$= 5\times10^{-6}\frac{d}{dt}\left[(4\sin 150t)\times10^{-3}\right] + 10^{-6}\frac{d}{dt}\left[(3\cos 150t)\times10^{-3}\right]$$

$$= (5\times10^{-6})(4\times10^{-3})\frac{d}{dt}(\sin 150t) + (10^{-6})(3\times10^{-3})\frac{d}{dt}(\cos 150t)$$

$$= (20\times10^{-9})\left[\cos 150t\right]150 + 3\times10^{-9}(-\sin 150t)150.$$

$$V_2 = (20\times10^{-9})\left[\cos 150t\right]150 + 3\times10^{-9}(-\sin 150t)150.$$

$$= \cancel{3000\times10^{-9}[\cos 150t] +}$$

$$=$$

$$= 150\times10^{-9}\left[20\cos 150t - 3\sin 150t\right]$$

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(4). An iron ring of 0.25 m diameter and $2\times10^{-3}$ sq.m. in cross section with saw-cut 1.5mm is wound with 150 turns of copper wire. The air gap flux density is ~~0.7 Telsa~~ 0.7 Tesla. The relative permeability of iron is 800. Calculate exciting current.

Sol: Given.

$D = 0.25\,m$ $\qquad [\because \mu_0 = 4\pi\times10^{-7}]$

$A = 2\times10^{-3}\,m^2$

$W = 1.5\,mm$

$N = 150$

$B = 0.7$ Tesla

$\mu_r = 800$

$I = ?$

$mmf = NI$

$$I = \frac{mmf}{N} \quad —(1).$$

But

w.k.T $\phi = \frac{mmf}{S}$

$mmf = \phi S$ $\qquad S = \frac{l}{\mu A} = \frac{\pi D}{\mu_0\mu_r A}$

$$= \frac{\pi\times0.25}{4\pi\times10^{-7}\times800\times2\times10^{-3}}$$

$$I = \frac{\phi S}{N} \quad —(2).$$

$$S = \frac{l}{\mu A} = \frac{\pi D}{\mu_0\mu_r A} = \frac{\pi\times0.25}{4\pi\times10^{-7}\times800\times2\times10^{-3}} = 390625$$

w.k.T

$$B = \frac{\phi}{A}$$

$\phi = BA$

$= 0.7\times2\times10^{-3} = 1.4\times10^{-3}$

sub $S$ & $\phi$ in eq—(2).

$$I = \frac{\phi S}{N} = \frac{1.4\times10^{-3}\times390625}{150} = 3.645$$

20/10/2023.

# UNIT:- II. Transient & steady state Analysis.

A network which contains storage elements like inductor and capacitor will take some amount of time to change its output when input change.

Transient state.

It is the state that a network undergoes before it reaches to constant value.

Transient time (or) transient response.

Time taken by the network to produce a constant response is said to be transient time.

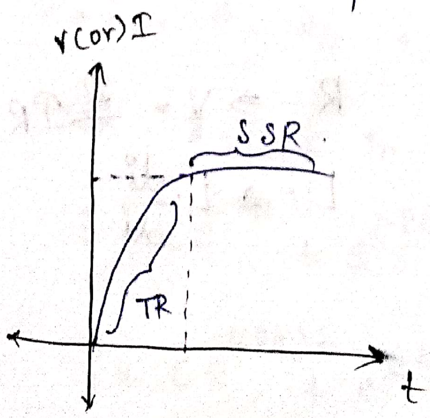
steady state.

It is the state that a network undergoes after it produces a constant value.

steady state response:

Time taken by the network to maintain constant response is said to be steady state response.

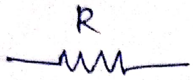
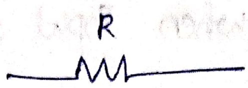
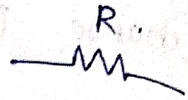
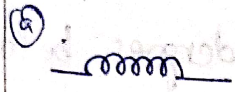
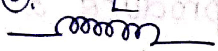
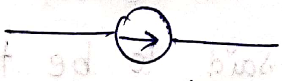
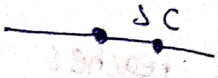
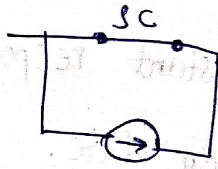
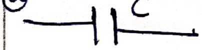
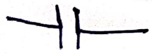
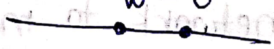
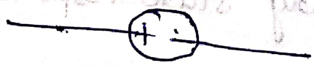
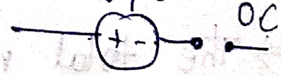
⇒ The total response of a network is the sumation of transient response and steady state response.




Total Response

= TR + SSR

Initial conditions (or) behaviour of base elements.

| S.NO. | Element. | Behaviour immediately after giving i/p (or) source (or) excitation. | Behaviour after giving i/p for long time ($t=\infty$). |
|---|---|---|---|
| 1. | R  | R  | R  |
| 2. | (a)  initial current $i_0 = 0$. <br> (b) L  $i_0 \neq 0$ | Inductor doesn't accept the sudden change in current. <br> O.C  $i_0 = 0$. <br>  $i_0 \neq 0$ | S C  $i_{at} \neq 0$ <br> S C  $i_{at}$ <br> $i_0 \neq 0$ |
| 3. | (a) C  initial voltage $V_0 = 0$ <br> (b)  $V_0 \neq 0$ | Capacitor doesn't accept the sudden change in voltage <br> $V_0 = 0$  <br> S.C  $V_0 \neq 0$ | $V_0 \neq 0$  O.C <br> $V_0 \neq 0$  OC |

$t = 0^- \rightarrow$ initial

$\boxed{t = 0 \rightarrow \text{giving i/p}}$

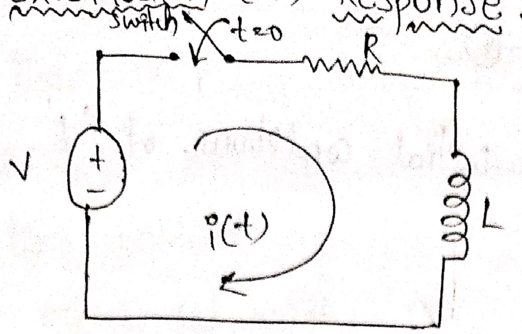
$t = 0^+ \rightarrow$ immediate

$t = \infty \rightarrow$ long time

$R \Rightarrow V = IR$

$L \Rightarrow L \cdot \frac{di}{dt}$

DC excitation (or) Response.




let us assume that.

1) Initial current flowing through the inductor is zero.

2) switch is closed at t=0.

Apply KVL, let $i(t) = i$.

$$V = iR + L\frac{di}{dt}$$

divide with 'L' on both sides.

$$\frac{V}{L} = \frac{R}{L}i + \frac{di}{dt}$$

$$\frac{di}{dt} + \frac{R}{L}i = \frac{V}{L} \longrightarrow (1).$$

standard 1st order differential eqn is.

$$\frac{dx}{dt} + Px = k \longrightarrow (2). \text{ where}$$

$$x = Ce^{-pt} + e^{-pt}\int ke^{pt}\,dt \longrightarrow (3).$$

Compare (1) & (2).

$$x = i, \quad P = R/L, \quad k = \frac{V}{L}.$$

sub. $x, P, k$ in eqn (3).

$$i = Ce^{(-R/L)t} + e^{(-R/L)t}\int \frac{V}{L}e^{(R/L)t}\,dt.$$

$$i = Ce^{(-R/L)t} + e^{(-R/L)t}\frac{V}{\cancel{L}}\frac{e^{(R/L)t}}{R} \times \cancel{L}$$

$$i = Ce^{(-R/L)t} + \frac{V}{R}\cancel{e^0}\,1$$

$$i = Ce^{(-R/L)t} + \frac{V}{R} \quad — (4).$$

To calculate. 'c' consider initial conditions. of 'L'.

1. initial current $i_0 = 0$.

2. at $t = 0^+$, $i = 0$.

substitute $t = 0$ & $i = 0$ in eq.$^n$ — (4).

$$0 = C\cancel{e^{(-R/L)0}}^{1} + \frac{V}{R}$$

$$\boxed{C = -\frac{V}{R}}$$

Sub 'c' in eq$^n$ — (4).

$$i = -\frac{V}{R}e^{(-R/L)t} + \frac{V}{R}$$

$$\boxed{i = \frac{V}{R}\left[1 - e^{(-R/L)t}\right]}$$

01/11/2023

Voltage across Resistor.
we know that.

$$V = IR$$

$$V_R = \frac{V}{\cancel{R}}\left[1 - e^{(-R/L)t}\right].\cancel{R}$$

$$\boxed{V_R = V\left[1 - e^{(-R/L)t}\right]} \quad — (5)$$

Voltage across inductor 'L'.

we know that, $V = L.\frac{di}{dt}$.

$$V_L = L.\frac{d}{dt}\left[\frac{V}{R}\left[1 - e^{(-R/L)t}\right]\right]$$

$$V_L = L.\frac{V}{R}\left[\cancel{\frac{d}{dt}(1)}^{0} - \frac{d}{dt}e^{(-R/L)t}\right]$$

$$V_L = \cancel{L}.\frac{V}{\cancel{R}}\left[-e^{(-R/L)t}\left(-\frac{\cancel{R}}{\cancel{L}}\right)\right]$$

$$\boxed{V_L = Ve^{(-R/L)t}} \quad — (6).$$

Time constant.

Def$^n$:- Time taken by voltage across the inductor to reach 36.8% of its stead state value, is said to be time constant.

It is represented with "$\tau$".

we know that.

$$V_L = V e^{(-R/L)t}$$

$$36.8\% \text{ of } V = V e^{(-R/L)t}$$

$$0.368 \cancel{V} = \cancel{V} e^{(-R/L)t}$$

Apply $\log_e$ on both sides.

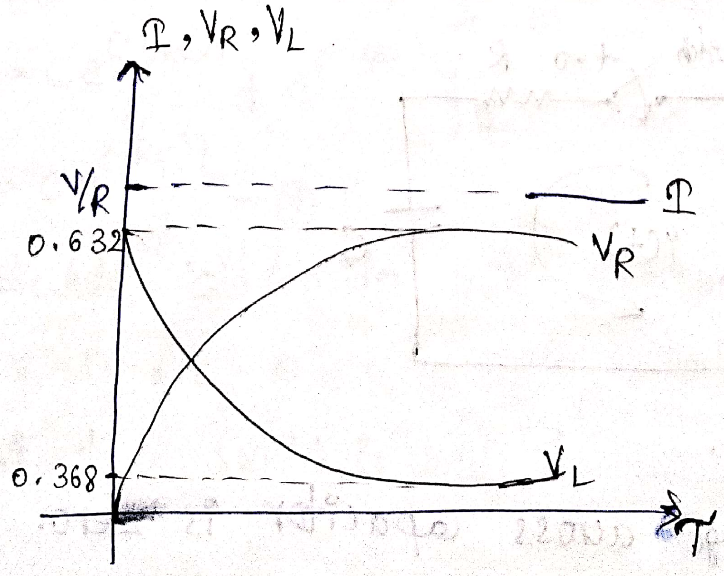
$$\log_e 0.368 = \log_e e^{(-R/L)t}$$

$$\cancel{-}0.99 = (\cancel{-}\frac{R}{L})t$$

let $t = \tau$

$$1 \simeq \frac{R}{L}\tau \Rightarrow \boxed{\tau = \frac{L}{R}}$$

The graphical relation of $\tau$ with respect to $I$, $V_R$, $V_L$ is as shown.

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Case (i) :- If $t = 0$.

$$i = \frac{V}{R}\left[1 - e^{(-R/L)t}\right] = \frac{V}{R}\left[1 - e^{(-R/L)0}\right] = 0$$

$$V_R = V\left[1 - e^{(-R/L)t}\right] = V\left[1 - e^{(-R/L)t}\right] = 0$$

$$V_L = V e^{(-R/L)t} = V$$

Case (ii) :- If $t = \infty$.

$e^{-\infty} = 0$
$e^{\infty} = 1/\infty$

$$i = \frac{V}{R}\left[1 - e^{(-R/L)t}\right] = \frac{V}{R}\left[1 - e^{-\infty}\right] = \frac{V}{R}$$

$$V_R = V\left[1 - e^{(-R/L)t}\right] = V\left[1 - e^{-\infty}\right] = V$$

$$V_L = V e^{(-R/L)t} = 0$$

Case (iii) :- If $t = \tau = \frac{L}{R}$.

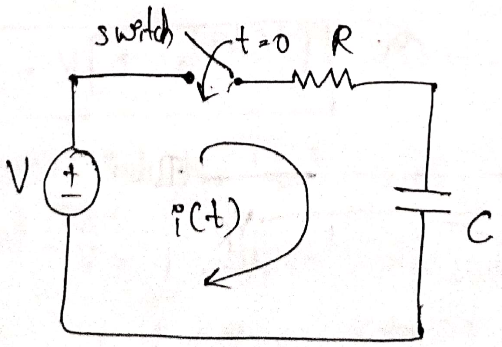
$e^{-1} =$

$$i = \frac{V}{R}\left[1 - e^{(-R/L)t}\right] = \frac{V}{R}\left[1 - e^{(-R/L)\frac{L}{R}}\right] = \frac{V}{R}\left[1 - e^{-1}\right] = 0.63$$

$$V_R = V\left[1 - e^{(-R/L)t}\right] = V\left[1 - e^{(-R/L)\frac{L}{R}}\right] = V\left[1 - e^{-1}\right] = 0.632$$

$$V_L = V e^{(-R/L)t} = V e^{(-R/L)\frac{L}{R}} = V e^{-1} = 0.368\,V$$

## DC excitation (or) DC response of series RC circuit.




Assume that.

1) Initial voltage across capacitor is zero.

2) Switch is closed at $t = 0$.

Apply KVL, Let $i(t) = i$

let $i(t) = i$

$V = iR + \frac{1}{C}\int i\,dt$

Apply $\frac{d}{dt}$ to above eqn.

$\cancel{\frac{dV}{dt}}^{0} = R\frac{di}{dt} + \frac{i}{C}$

$0 = R\frac{di}{dt} + \frac{i}{C}$

divide above eqn with 'R'.

$\frac{di}{dt} + \frac{1}{RC}i = 0$ — ①.

Standard 1st order differential eqn.

$\frac{dx}{dt} + Px = k$ — ② where

$x = ce^{-Pt} + e^{-Pt}\int ke^{Pt}\,dt$ — ③.

Compare ① & ②.

$x = i$, $P = \frac{1}{RC}$, $k = 0$.

Sub. $x, P, k$ in eqn — ③.

$i = ce^{(-1/RC)t} + \cancel{e^{(-1/RC)t}\int 0}^{0}$

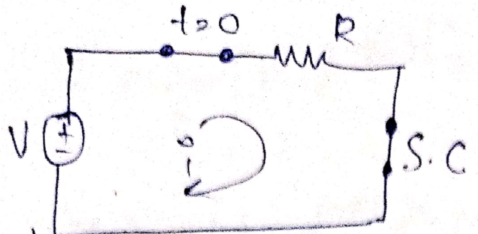
$i = ce^{(-1/RC)t}$ — ④

To calculate 'c' consider initial conditions of 'c'.

(i) $V = 0$, $t = 0^-$

(ii) at $t = 0$, switch is closed.

Redraw the ckt.




Capacitor is 'SC' for $V=0$.

Apply KVL

$V = iR$

$\boxed{i = \frac{V}{R}}$ for $t = 0^-$ (or) $t = 0^+$

Substitute $i = \frac{V}{R}$ and $t = 0$ in ④.

$\frac{V}{R} = C e^{0} \to 0$ $\quad \frac{V}{R} = C e^{-0} \to 1$

$\boxed{C = \frac{V}{R}}$

Substitute 'C' in eqn – ④.

$$\boxed{i = \frac{V}{R} e^{(-1/RC)t}}$$

Voltage across Resistor.

we know that

$V = IR$

$V_R = \left[\frac{V}{R} e^{(-1/RC)t}\right] R$

$$\boxed{V_R = V e^{(-1/RC)t}}$$

03-11-2023.

Voltage across Capacitor.

$V = \frac{1}{C}\int i\,dt$

$V_c = \frac{1}{C}\int i\,dt$

$= \frac{1}{C}\int \frac{V}{R} ($

$= \frac{1}{C}\int \frac{V}{R} e^{(-1/RC)t} dt$

$$= \frac{V}{RC} \int e^{(-1/RC)t}\, dt$$

$$= \frac{V}{RC} \frac{e^{(-1/RC)t}}{-1/RC} + C$$

$$= \frac{V}{RC} \frac{e^{(-1/RC)t}}{-1/RC} + C$$

$$V_C = -Ve^{(-1/RC)t} + C$$

To calculate 'c' consider initial conditions of 'τ'.

1) $V_c = 0$ for $t = 0^-$.

Sub $V_c = 0$ & $t = 0$ in above eq$^n$.

$$0 = -V\cancel{e}^{\,1} + C$$

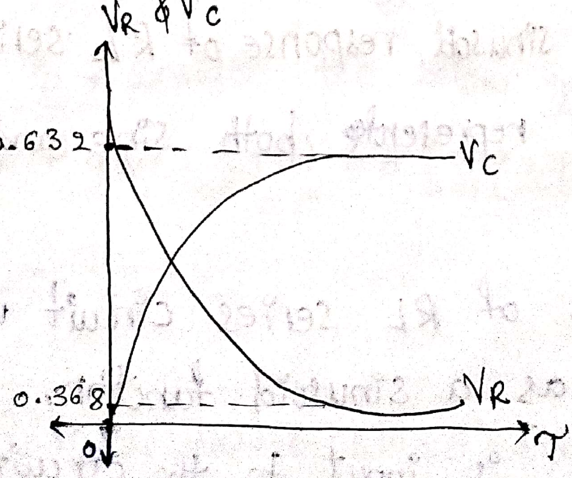
$$C = V$$

Sub 'C' in $V_c$.

$$V_c = -Ve^{(-1/RC)t} + V$$

$$\boxed{V_c = V\left[1 \cdot e^{(-1/RC)t}\right]}$$

⇒ The graphical relation of time constant 'τ' w.r.t $V_c$ & $V_R$ is as shown.



Time constant of RC series circuit.

Defⁿ: Time taken by voltage across the capacitor to reach 63.2% of its steady state value, is said to be Time constant of RC circuit.

⇒ It is Represented with '$\tau$'.

we know that

$$V_C = V\left[1 - e^{(-1/RC)t}\right]$$

$$63.2\% \text{ of } V = V\left[1 - e^{(-1/RC)t}\right]$$

$$0.632\cancel{V} = \cancel{V}\left[1 - e^{(-1/RC)t}\right]$$

$$e^{(-1/RC)t} = 1 - 0.632$$

$$e^{(-1/RC)t} = 0.368$$

Apply $\log_e$ on Both sides.

$$\log_e e^{(-1/RC)t} = \log_e 0.368$$

$$(\cancel{-}1/RC)t = \cancel{-}0.99$$

if $t = \tau$

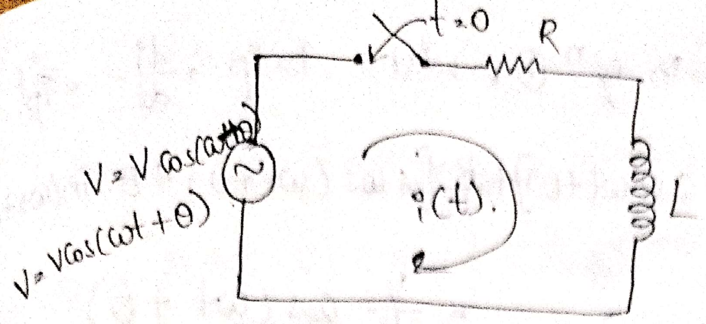
$$\frac{\tau}{RC} \simeq 1 \Rightarrow \boxed{\tau = RC}$$

AC response (or) sinusoid response of RL series circuit.

⇒ sinusoid function represents both sine and cosine signals.

⇒ for the analysis of RL series circuit we consider cosine signal as a sinusoid function.

⇒ and the same is input to the circuit.




let us assume that.
condition.

1) Initial current flowing through the inductor is zero.

2) Switch is closed at $t=0$.

let

Apply KVL, Let $i(t) = i$.

$$V = iR + L\frac{di}{dt}$$

$$V\cos(\omega t+\theta) = iR + L\frac{di}{dt}.$$

divide the $eq^n$ with 'L' on both sides.

$$\frac{di}{dt} + \frac{R}{L}i = \frac{V}{L}\cos(\omega t+\theta) \quad \text{—} (1).$$

solution for above $eq^n$ is 'i'.

$$i = i_c + i_p \quad \text{—} (2).$$ Here $i_c$ = Complementary solution.
$i_p$ = particular solution.

$$i_c = C e^{(-R/L)t} \quad \text{—} (3).$$

To find $i_p$, consider the below $eq^n$.

$$i_p = A\cos(\omega t+\theta) + B\sin(\omega t+\theta) \quad \text{—} (4).$$

$$\cancel{i_p' = \frac{d}{dt} i_p = -A\omega\cos}$$

$$i_p' = \frac{d}{dt} i_p = -A\omega\sin(\omega t+\theta) + B\omega\cos(\omega t+\theta) \quad \text{—} (5).$$

sub $i_p$ & $i_p'$ in eq$^n$ ①, where $i = i_p$, $\frac{di}{dt} = i_p'$.

$$\left[-A\omega \sin(\omega t+\theta) + B\omega \cos(\omega t+\theta)\right] + \frac{R}{L}\left[A \cos(\omega t+\theta) + B\sin(\omega t+\theta)\right]$$

$$= \frac{V}{L}\cos(\omega t+\theta).$$

Common terms separate.

$$\sin(\omega t+\theta)\left[-A\omega + \frac{BR}{L}\right] + \cos(\omega t+\theta)\left[B\omega + \frac{RA}{L}\right] = \frac{V}{L}\cos(\omega t+\theta)$$

Compare LHS & RHS.

$$-A\omega + \frac{BR}{L} = 0 \rightarrow ⓐ$$

$$A\omega = \frac{BR}{L}$$

$$A = \frac{BR}{\omega L} \text{ — ⓒ}.$$

$$B\omega + \frac{RA}{L} = \frac{V}{L} \text{ — ⓑ}.$$

sub ⓒ in ⓑ.

$$B\omega + \frac{R}{L}\cdot\frac{BR}{\omega L} = \frac{V}{L}$$

$$B\left[\omega + \frac{R^2}{L^2\omega}\right] = \frac{V}{L}$$

$$B\left[\frac{\omega^2 L^2 + R^2}{L^2\omega}\right] = \frac{V}{L}$$

$$B = \frac{V}{\cancel{L}}\cdot\frac{L^{\cancel{2}}\omega}{\omega^2 L^2 + R^2}$$

$$\boxed{B = \frac{VL\omega}{\omega^2 L^2 + R^2}}$$

Now 'B' in eq$^n$ ⓒ.

$$A = \frac{R}{\omega \cancel{L}}\cdot\frac{V\cancel{L}\cancel{\omega}}{\omega^2 L^2 + R^2}$$

$$\boxed{A = \frac{RV}{\omega^2 L^2 + R^2}}$$

Substitute A & B in eq$^n$ ④.

$$i_p = A\cos(\omega t+\theta) + B\sin(\omega t+\theta)$$

$$i_p = \frac{RV}{\omega^2 L^2 + R^2}\cos(\omega t+\theta) + \frac{VL\omega}{\omega^2 L^2 + R^2}\sin(\omega t+\theta)$$

let us assume.

$$M\cos\phi = \frac{RV}{\omega^2 L^2 + R^2}, \quad M\sin\phi = \frac{VL\omega}{\omega^2 L^2 + R^2}$$

$$i_p = M\cos\phi\cos(\omega t+\theta) + M\sin\phi\sin(\omega t+\theta)$$

$$= M[\cos(\omega t+\theta)\cos\phi + \sin(\omega t+\theta)\sin\phi]$$

$$i_p = M\cos(\omega t+\theta-\phi) \quad \text{—} (6)$$

To calculate M. → squarring and adding the assumed terms.

$$M^2\cos^2\phi + M^2\sin^2\phi = \frac{R^2V^2}{(\omega^2L^2+R^2)^2} + \frac{V^2L^2\omega^2}{(\omega^2L^2+R^2)^2}$$

$$M^2[\cos^2\phi + \sin^2\phi] = \frac{V^2[R^2+\omega^2L^2]}{[\omega^2L^2+R^2]^2}$$

$$M^2 = \frac{V^2}{\omega^2L^2+R^2}$$

$$\boxed{M = \frac{V}{\sqrt{\omega^2L^2+R^2}}}$$

To calculate $\phi$

→ Dividing the assumed terms.

$$\frac{M\sin\phi}{M\cos\phi} = \tan\phi$$

$$\tan\phi = \frac{\cancel{V}L\omega}{\cancel{\omega^2L^2+R^2}} \times \frac{\cancel{\omega^2L^2+R^2}}{R\cancel{V}}$$

$$\tan\phi = \frac{\omega L}{R}$$

$$\boxed{\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)}$$

$$i_p = \frac{V}{\sqrt{\omega^2L^2+R^2}}\cos\left(\omega t+\theta-\tan^{-1}\left(\frac{\omega L}{R}\right)\right) \quad \text{—} (7)$$

sub eqⁿ (3) & (7) in eqⁿ (2).

$i = i_c + i_p$

$$i = Ce^{(-R/L)t} + \frac{V}{\sqrt{\omega^2L^2+R^2}} \cos\left(\omega t + \theta - \tan^{-1}\left(\frac{\omega L}{R}\right)\right) \quad -(8)$$

To calculate 'c', consider initial conditions of 'L'.

(i) $i = 0$ for $t = 0^-$

sub $i = 0$ & $t = 0$ in eq$^n$-(8).

$$0 = C\cancel{e^{0}}^{1} + \frac{V}{\sqrt{\omega^2L^2+R^2}} \cos\left(\theta - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)$$

$$C = \frac{-V}{\sqrt{\omega^2L^2+R^2}} \cos\left(\theta - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)$$

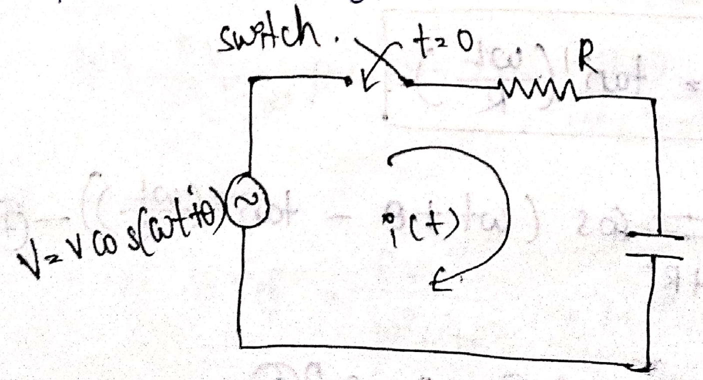
sub C in eq$^n$ (8).

$$i = \frac{-V}{\sqrt{\omega^2L^2+R^2}} \cos\left(\theta - \tan^{-1}\left(\frac{\omega L}{R}\right)\right) e^{(-R/L)t} + \frac{V}{\sqrt{\omega^2L^2+R^2}} \cos\left(\omega t + \theta - \tan^{-1}(\omega$$

AC Response (or) Sinusoid response of RC series circui

⇒ sinusoid function represents both sine and cosine signals.

⇒ for the analysis of RC series circuit we consider Cosine signal as a sinusoid function. and the same is input to the circuit.



Let us assume that

conditions.

1) Initial current flowing through the capacitor is zero.

2) switch is closed at $t=0$.

Apply KVL, let $i(t)=i$

$$V = iR + \frac{1}{C}\int i\,dt$$

$$V\cos(\omega t+\theta) = iR + \frac{1}{C}\int i\,dt$$

Apply $\frac{d}{dt}$ on Both sides.

$$-V\omega\sin(\omega t+\theta) = R\frac{di}{dt} + \frac{i}{C}$$

divide $eq^n$ with 'R'.

$$\frac{di}{dt} + \frac{i}{RC} = \frac{-V\omega}{R}\sin(\omega t+\theta) \rightarrow (1)$$

solution to $eq^n$ (1) is 'i'.

$$i = i_c + i_p \rightarrow (2).$$

$i_c$ = Complementary solution

$$i_c = ce^{(-1/RC)t} \rightarrow (3).$$

$i_p$ = particular solution.

To calculate $i_p$, consider following $eq^n$.

$$i_p = A\cos(\omega t+\theta) + B\sin(\omega t+\theta) \rightarrow (4)$$

$$i_p' = \frac{di_p}{dt} = -A\omega\sin(\omega t+\theta) + B\omega\cos(\omega t+\theta) \rightarrow (5)$$

Substitute 4,5 in $eq^n$-(1), where $i = i_p$, $\frac{di}{dt} = i_p'$,

$$\left[-A\omega\sin(\omega t+\theta) + B\omega\cos(\omega t+\theta)\right] + \frac{1}{RC}\left[A\cos(\omega t+\theta) + B\sin(\omega t+\theta)\right] = \frac{-V\omega}{R}\sin(\omega t+\theta).$$

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Common terms separate.

$$\sin(\omega t+\theta)\left[-A\omega+\frac{B}{RC}\right]+\cos(\omega t+\theta)\left[B\omega+\frac{A}{RC}\right]=\frac{-V\omega}{R}\sin(\omega t+\theta)$$

Comparing LHS & RHS.

$$-A\omega+\frac{B}{RC}=\frac{-V\omega}{R} \quad \text{—(a)}$$

$$B\omega+\frac{A}{RC}=0 \quad \text{—(b)}$$

$$B\omega=\frac{-A}{RC}$$

$$\boxed{B=\frac{-A}{\omega RC}} \quad \text{—(c)}$$

Sub (c) in (a).

$$-A\omega+\frac{1}{RC}\left[\frac{-A}{\omega RC}\right]=\frac{-V\omega}{R}$$

$$\cancel{-}A\left[\omega+\frac{1}{\omega R^2C^2}\right]=\frac{\cancel{-}V\omega}{R}$$

$$A\left[\frac{\omega^2R^2C^2+1}{\omega^2R^2C^2}\right]=\frac{V\omega}{R}$$

$$A=\frac{V\omega}{\cancel{R}}\times\frac{\omega R^{\cancel{2}}C^2}{1+\omega^2R^2C^2}=$$

$$A=\frac{V\omega^2C^2R}{1+\omega^2R^2C^2}=\frac{V\cancel{\omega^2}\cancel{C^2}R}{\cancel{\omega^2}\cancel{C^2}\left[R^2+\frac{1}{\omega^2C^2}\right]}$$

$$\boxed{A=\frac{VR}{R^2+\frac{1}{\omega^2C^2}}}$$

Sub eqn A in eqn (c)

$$B=\frac{-1}{\omega RC}\cdot\frac{VR}{R^2+\frac{1}{\omega^2C^2}}$$

$$\boxed{B=\frac{-V}{\omega C\left[R^2+\frac{1}{\omega^2C^2}\right]}}$$

Substitute A & B in eqn (4).

$$i_p=\frac{V_R}{R^2+\frac{1}{\omega^2C^2}}\cos(\omega t+\theta)-\frac{V}{\omega C\left[R^2+\frac{1}{\omega^2C^2}\right]}\sin(\omega t+\theta)$$

let $M\cos\phi=\dfrac{V_R}{R^2+\frac{1}{\omega^2C^2}}$, $M\sin\phi=\dfrac{V}{\omega C\left[R^2+\frac{1}{\omega^2C^2}\right]}$

$$i_p=M\cos\phi\cos(\omega t+\theta)-M\sin\phi\sin(\omega t+\theta).$$

$$i_p=M\left[\cos(\omega t+\theta)\cos\phi-\sin(\omega t+\theta)\sin\phi\right]$$

$\therefore \cos A\cos B-\sin A\sin B=\cos(A+B)$

$$i_p = M\cos((\omega t+\theta)+\phi)$$

$\Rightarrow$ To calculate $M$.

$$M^2\cos^2\phi + M^2\sin^2\phi = \frac{V^2R^2}{\left(R^2+\frac{1}{\omega^2C^2}\right)^2} + \frac{V^2}{\omega^2C^2\left[R^2+\frac{1}{\omega^2C^2}\right]^2}$$

$$M^2\left[\cos^2\phi+\sin^2\phi\right] = \frac{V^2}{\left(R^2+\frac{1}{\omega^2C^2}\right)^{\cancel{2}}}\left[\cancel{R^2+\frac{1}{(\omega C)^2}}\right]$$

$$M^2 = \frac{V^2}{R^2+\frac{1}{\omega^2C^2}}$$

$$\boxed{M = \frac{V}{\sqrt{R^2+\frac{1}{\omega^2C^2}}}}$$

$\Rightarrow$ To find $\phi$

$$\frac{M\sin\phi}{M\cos\phi} = \tan\phi$$

$$\tan\phi = \frac{\cancel{V}}{\omega C\left(R^2+\frac{1}{(\omega C)^2}\right)} \times \frac{\left(R^2+\frac{1}{(\omega C)^2}\right)}{\cancel{V}R}$$

$$\tan\phi = \frac{1}{\omega RC}$$

$$\boxed{\phi = \tan^{-1}\left(\frac{1}{\omega RC}\right)}$$

$$i_p = \frac{V}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}}\cos\left(\omega t+\theta+\tan^{-1}\left(\frac{1}{\omega RC}\right)\right) \quad — (7)$$

sub (3) & (7) in (2).

$$i = Ce^{(-1/RC)t} + \frac{V}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}}\cos\left[\omega t+\theta+\tan^{-1}\left(\frac{1}{\omega RC}\right)\right] \quad — (8)$$

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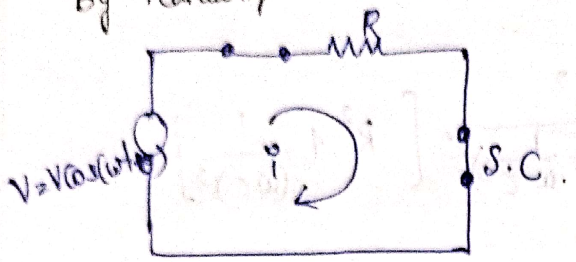
To calculate 'c' consider initial conditions of 'c'.

i) $V=0,\ t=0^-$

ii) at $t=0$, switch is closed.

By redrawing the ckt with $t=0$.




Apply KVL.

$$V = iR$$

$$V\cos(\omega t+\theta) = iR$$

$$\boxed{i = \frac{V\cos(\omega t+\theta)}{R}}$$ for $t=0$.

Substitute $t$ & $i=0$ in eqⁿ ⑧.

$$\frac{V\cos(\omega t+\theta)}{R} = C\cancelto{1}{e^{0}} + \frac{V}{\sqrt{R^2+\frac{1}{\omega^2C^2}}}\cos\left(0+\theta+\tan^{-1}\left(\frac{1}{\omega RC}\right)\right)$$

$$C = \frac{V\cos(\omega t+\theta)}{R} - \frac{V}{\sqrt{R^2+\frac{1}{\omega^2C^2}}}\cos\left(\theta+\tan^{-1}\left(\frac{1}{\omega RC}\right)\right)$$

Substitute C in eqⁿ ⑧

$$i = \frac{V\cos(\omega t+\theta)}{R} - \frac{V}{\sqrt{R^2+\frac{1}{\omega^2C^2}}}\cos\left(\theta+\tan^{-1}\left(\frac{1}{\omega RC}\right)\right)e^{(-1/RC)t} + \frac{V}{\sqrt{R^2+\frac{1}{\omega^2C^2}}}\cos\left[\omega t+\theta+\tan^{-1}\left(\frac{1}{\omega RC}\right)\right]$$

## Step Response of series RC circuit

The Response of a RC circuit, when step signal is given as an input is said to be Step Response of RC circuit.

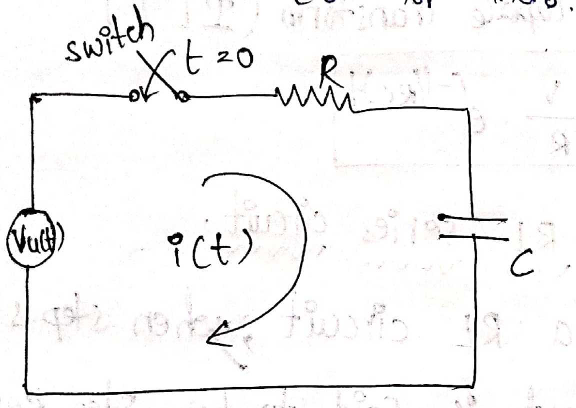
⇒ The step signal is represented with $u(t)$.

⇒ If the amplitude is one.

⇒ step signal with amplitude 'V' is represented with $Vu(t)$

Mathematical expression for $V.u(t)$ is given by.

$$Vu(t) = \begin{cases} V & \text{for } t \geq 0 \\ 0 & \text{for others.} \end{cases}$$




let us assume that

Initial Conditions.

1) Initial ~~Volt~~ Voltage across Capacitor is '0'.

2) Switch is closed at $t = 0$.

Apply KVL

$$Vu(t) = i(t)R + \frac{1}{C}\int i(t)\,dt$$

Apply Laplace Transform (L.T).

$$V\,LT\{u(t)\} = R\,LT\{i(t)\} + \frac{1}{C}LT\{\int i(t)\,dt\}$$

$$\frac{V}{s} = RI(s) + \frac{1}{C}\cdot\frac{I(s)}{s}$$

$$\frac{V}{s} = I(s)\left[R + \frac{1}{Cs}\right]$$

$$I(s) = \frac{V}{s\left[R + \frac{1}{Cs}\right]}$$

if $x(t) \overset{L.T}{\longleftrightarrow} X(s)$ then

$\int x(t)\,dt \overset{L.T}{\longleftrightarrow} \frac{X(s)}{s}$

$\frac{d}{dt}x(t) \overset{L.T}{\longleftrightarrow} X(s).s$

$u(t) \overset{L.T}{\longleftrightarrow} \frac{1}{s}$

$e^{-at} \overset{L.T}{\longleftrightarrow} \frac{1}{s+a}$

$$I(s) = \frac{V}{s\left[R + \frac{1}{Cs}\right]}$$

$$I(s) = \frac{VC\cancel{s}}{\cancel{s}[RCs+1]}$$

$$I(s) = \frac{VC}{RCs+1}$$

$$I(s) = \frac{V\cancel{C}}{R\cancel{C}\left[s+\frac{1}{RC}\right]} \Rightarrow I(s) = \frac{V}{R} \cdot \frac{1}{s+\frac{1}{RC}}$$

Apply Inverse Laplace Transform (ILT)

$$\boxed{i(t) = \frac{V}{R} \cdot e^{(-1/RC)t}}$$

## Step Response of R.L series circuit.

⇒ The Response of a RL circuit, when step signal is given as an input is said to be step Response of RL circuit.

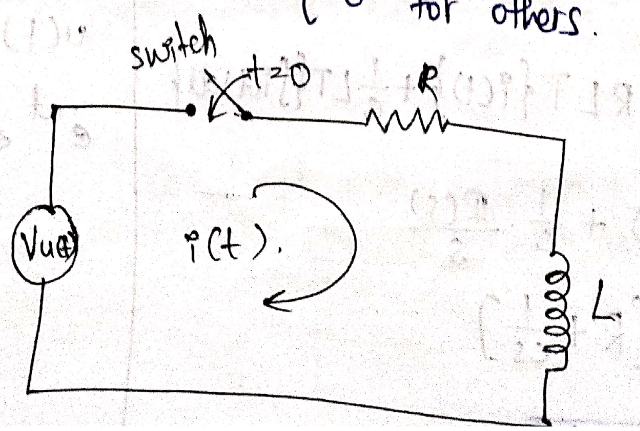
⇒ The step signal is represented with $u(t)$.

⇒ If the amplitude is '1'.

⇒ Step signal with amplitude 'V' is represented with $Vu(t)$.

Mathematical expression for $Vu(t)$ is given by

$$Vu(t) = \begin{cases} V & \text{for } t \geq 0 \\ 0 & \text{for others.} \end{cases}$$



Let us assume that

Initial Conditions.

1) Initial voltage across Inductor is '0'.
2) switch is closed at $t = 0$.

Apply KVL.

$$Vu(t) = i(t)R + L\frac{di(t)}{dt}$$

Apply L.T.

$$V\,LT\{u(t)\} = R\,LT\{i(t)\} + L\cdot LT\left\{\frac{di(t)}{dt}\right\}$$

$$\frac{V}{s} = R\cdot I(s) + L\cdot I(s)\cdot s$$

$$\frac{V}{s} = I(s)\,[R + Ls]$$

$$I(s) = \frac{V}{s[R+Ls]}$$

$$I(s) = \frac{V}{s[L[s+\frac{R}{L}]]}$$

I(s) can be written as.

$$I(s) = \frac{V}{\cancel{L}}\cdot\frac{\cancel{L}}{R}\left[\frac{1}{s} - \frac{1}{s+\frac{R}{L}}\right]$$

$$I(s) = \frac{V}{R}\left[\frac{1}{s} - \frac{1}{s+R/L}\right]$$

Apply ILT.

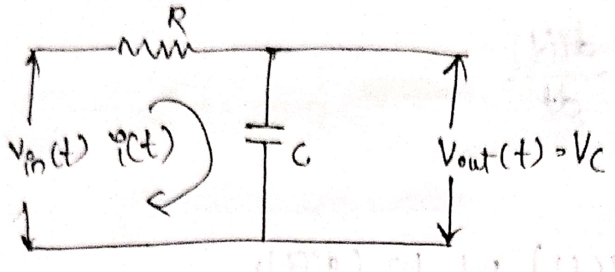
$$\boxed{\begin{aligned} i(t) &= \frac{V}{R}\left[1 - e^{(-R/L)t}\right] \\ &\text{(or)} \\ &= \frac{V}{R}\left[u(t) - e^{(-R/L)t}\right] \end{aligned}}$$

# RC circuit as an Integrator.

06/11/2023.

A Low-pass RC circuit can act as an Integrator.

⇒ An RC circuit whose (or) time constant is much more higher than the time period of input signal ($RC >> T$) is called as low-pass RC circuit.




1) for $RC >> T$, voltage across the capacitor will take large time to reach its maximum value (or) Capacitor takes long time to get charged. And hence the entire input voltage appear across resistor.

$$V_R = V_{in}(t).$$

$V_R \rightarrow$ voltage across the resistor.

2) as $V_R = V_{in}(t)$, the current in the circuit is due to resistor (R) and is given by.

$$i = \frac{V_R}{R} = \frac{V_{in}(t)}{R}.$$

3) from the circuit, it can be observed that output voltage is equal to voltage across the capacitor.

$$V_{out} = V_C$$

$$V_{out} = V_C = \frac{1}{C}\int i\,dt$$

$$= \frac{1}{C}\int \frac{V_{in}(t)}{R}\,dt \quad [\because \text{from point 2}].$$

$$V_{out}(t) = \frac{1}{RC}\int V_{in}(t)\,dt$$

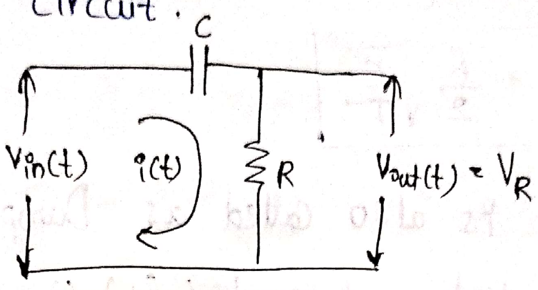
$$V_{out}(t) \propto \int V_{in}(t).$$

Hence it can be concluded that as output voltage is directly proportional to integr of input voltage, a low-pass RC ckt can be act as an Integrator.

## RC circuit as a Differentiator.

A High-pass RC circuit can act as a Differentiator.

⇒ An RC circuit whose time constant is very less than the the time period of input signal ($R_C << T$) is called as High-pass RC circuit.




1) for $RC << T$, voltage across the capacitor gets charged quickly. And hence the entire input voltage appear across capacitor.

$$V_C = V_{in}(t).$$ $V_C \rightarrow$ Voltage across the capacitor.

2) As $V_C = V_{in}(t)$, the current in the circuit is due to capacitor (c) and is given by.

$$i = C\frac{dV_C}{dt} = C\cdot\frac{dV_{in}(t)}{dt}.$$

3) From the circuit, it can be observed that output voltage is equal to voltage across the resistor.

$$V_{out}(t) = V_R = iR$$

$$V_{out}(t) = C\frac{dV_{in}(t)}{dt}R$$

$$V_{out}(t) = RC\frac{d}{dt}(V_{in}(t))$$

$$V_{out}(t) \propto \frac{d}{dt}V_{in}(t).$$

∴ Hence it can be concluded that as output voltage is directly proportional to the differention of input voltage, a high-pass RC ckt can be act as a differentiator.

# Damping factor

Defⁿ :- It is defined as the ratio of damping coefficient to critically damping coefficient.

⇒ It is repesented with zeeta ($\zeta$).

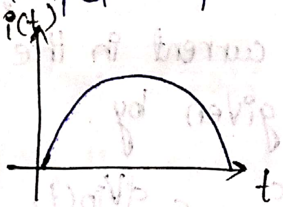
$$\zeta = \frac{C}{C_c}$$

$$\boxed{\zeta = \frac{R}{2}\sqrt{\frac{C}{L}}}$$

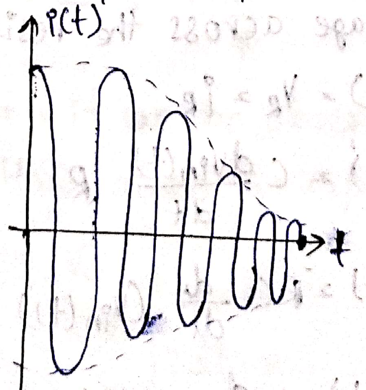
Damping factor is also called as Damping Ratio.

⇒ Based on the value of zeeta ($\zeta$), the response of circuit is categorized into 3 cases.

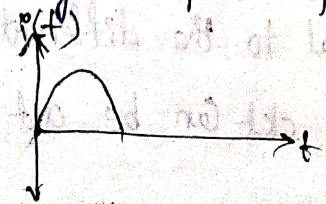
Case (i) :- If $\zeta > 1$, then the response of the circuit is called overdamped response.



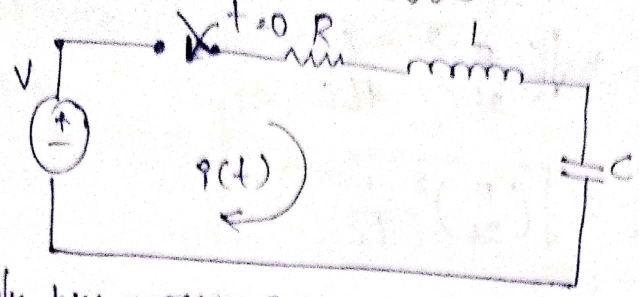

Case (ii) :- If $\zeta < 1$, then the response of the circuit is called underdamped response.




Case (iii) :- If $\zeta = 1$, then the response of the circuit is called critically damped response.



## DC Response of series RLC circuit (or) Second order series RLC circuit.




Apply KVL, $i(t) = i$

$$V = iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$$

Apply $\frac{d}{dt}$ on B.S

$$\frac{dV}{dt} = R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{C}$$

$$0 = L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{i}{C}$$

divide eq$^n$ with 'L'.

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0$$

$$i\left[\frac{d^2}{dt^2} + \frac{R}{L}\frac{d}{dt} + \frac{1}{LC}\right] = 0 \quad -①$$

eq$^n$-① can be written as. $\therefore \left(\frac{d}{dt} = D\right)$.

**
$$D^2 + \frac{R}{L}D + \frac{1}{LC} = 0 \quad -②$$

We know that. for linear eq$^n$ $ax^2 + bx + c = 0$

roots,

$$x_1, x_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Similarlly roots of eq$^n$-② can be.

$$D_1, D_2 = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}}{2}$$

$$D_1, D_2 = \frac{-R}{2L} \pm \frac{1}{2}\sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}$$

$$D_1, D_2 = \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{4}{4LC}}$$

$$D_1, D_2 = \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

if $k_1 = \frac{-R}{2L}$, $k_2 = \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$

$D_1 = k_1 + k_2$, $D_2 = k_1 - k_2$

08/11/2023

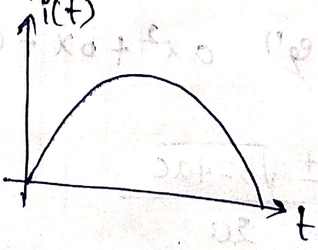
from the above equations it can be observed that $k_1$ as -ve polarity & the polarity of $k_2$ depends on $\frac{R}{2L}$ and $\frac{1}{LC}$.

⇒ The nature of the roots and solution i for second order differential equation can be obtained in following 3 cases:-

Case (i) :- if $\frac{R}{2L} > \frac{1}{LC}$

⇒ The roots are real and unequal.

⇒ for $\frac{R}{2L} > \frac{1}{LC}$ the response the circuit is called as overdamped response.




⇒ Solution 'i' for overdamped response of RLC series circuit is

$$\boxed{i = C_1 e^{D_1(t)} + C_2 e^{D_2(t)}}$$

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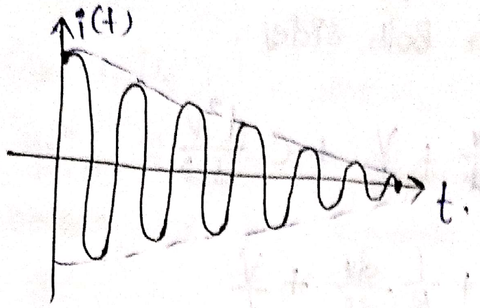
Case (ii) :- $\frac{R}{2L} < \frac{1}{LC}$

⇒ The roots are complex conjugate.

⇒ The response of the circuit is called as under damped response.



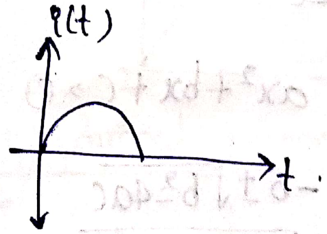

⇒ The solution 'i' for undamped response of RLC series circuit is given by.

$$\boxed{i = e^{k_1(t)}[C_1 \cos k_2(t) + C_2 \sin k_2(t)]}$$

Case (iii) :- if $\frac{R}{2L} = \frac{1}{LC}$

⇒ The roots are real and equal.

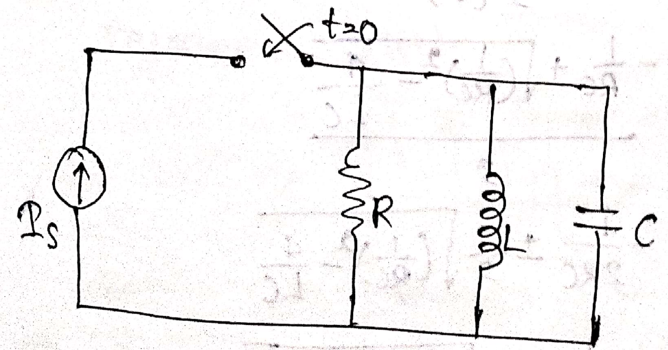
⇒ The response of the circuit is called as critically damped response.




⇒ solution 'i' for critically damped response of RLC series circuit is given by

$$\boxed{i = e^{k_1(t)}[C_1 + C_2(t)]}$$

Second order parallel RLC circuit.



Apply KCL .

$$I_s = \frac{V}{R} + \frac{1}{L}\int v\,dt + C\frac{dv}{dt}$$

Apply $\frac{d}{dt}$ on Both sides .

$$\frac{dI_s}{dt} = \frac{1}{R}\frac{dv}{dt} + \frac{V}{L} + C\frac{d^2v}{dt^2}$$

$$0 = C\frac{d^2v}{dt^2} + \frac{1}{R}\frac{dv}{dt} + \frac{V}{L}$$

divide eq$^n$ with 'c'.

$$\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{V}{LC} = 0.$$

$$V\left[\frac{d^2}{dt^2} + \frac{1}{RC}\frac{d}{dt} + \frac{1}{LC}\right] = 0 \quad \text{—①}.$$

eq$^n$ ① Can be written as .

$$D^2 + \frac{1}{RC}D + \frac{1}{LC} = 0. \quad \text{—②}$$

WKT

for linear eq$^n$ $ax^2+bx+c=0$

roots .

$$x_1, x_2 = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Similarly roots of eq$^n$ — ② Can be .

$$D_1, D_2 = \frac{-\frac{1}{RC} \pm \sqrt{\left(\frac{1}{RC}\right)^2 - 4(1)\left(\frac{1}{LC}\right)}}{2(1)}$$

$$= \frac{-\frac{1}{RC} \pm \sqrt{\left(\frac{1}{RC}\right)^2 - \frac{4}{LC}}}{2}$$

$$= -\frac{1}{2RC} \pm \frac{1}{2}\sqrt{\left(\frac{1}{RC}\right)^2 - \frac{4}{LC}}$$

$$= -\frac{1}{2RC} \pm \sqrt{\frac{1}{(2RC)^2} - \frac{4}{4LC}}$$

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$$D_1, D_2 = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

$$k_1 = -\frac{1}{2RC}, \quad k_2 = \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

$$\cdot D_1 = k_1 + k_2, \quad D_2 = k_1 - k_2$$

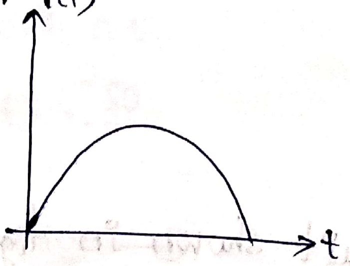
from the above eq$^n$s it can be observed that $k_1$ as -ve polarity & the polarity of $k_2$ depends on $\frac{1}{2RC}$ and $\frac{1}{LC}$.

→ The nature of the roots and solution; for second order differential equation can be obtained in following 3 cases.

case (i) :- if $\frac{1}{2RC} > \frac{1}{LC}$

⇒ The roots are real and unequal.

⇒ for $\frac{1}{2RC} > \frac{1}{LC}$ the response of the circuit is called as overdamped response.

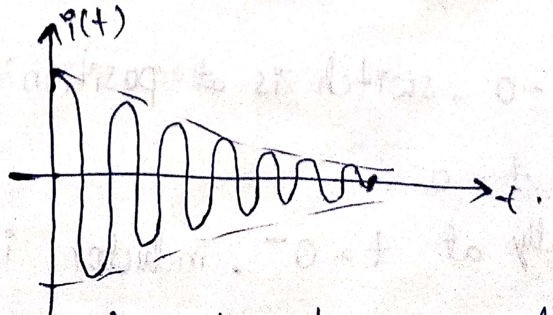



→ solution 'i' for overdamped response of RLC parallel circuit is. $\boxed{i = C_1 e^{D_1(t)} + C_2 e^{D_2(t)}}$

case (ii) :- $\frac{1}{2RC} < \frac{1}{LC}$.

⇒ The roots are complex conjugate.

⇒ The response of the circuit is called as underdamped response.



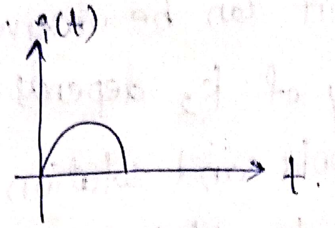

⇒. The solution 'i' for undamped response of RLC parallel circuit is given by

$i = e^{k_1(t)}[c_1 \cos k_2(t) + c_2 \sin k_2(t)]$.

Case (iii) :- $\frac{1}{2RC} = \frac{1}{LC}$

⇒ The real roots are real and equal.

⇒ The response of the circuit is called as critically damped response.

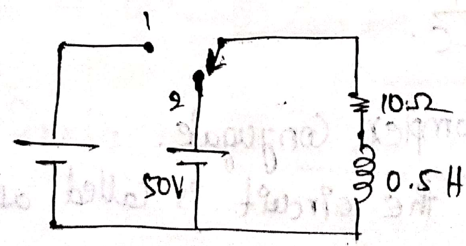



⇒ Solution 'i' for critically damped response of RLC parallel circuit is given by.

$$\boxed{i = e^{k_1(t)}[c_1 + c_2(t)]}$$

Problems :-

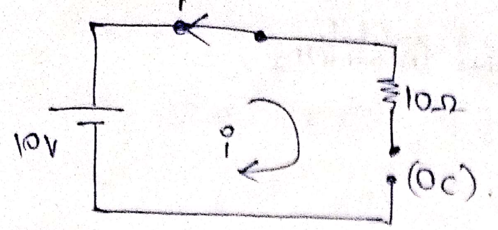
1) In the circuit shown in figure. Determine the current i(t) when switch is change from position 1 to position 2. The switch is moved from 1 to 2 at t = 0.




Sol :- At t = 0, switch is at position '2' — i(t).

$t = 0^-$

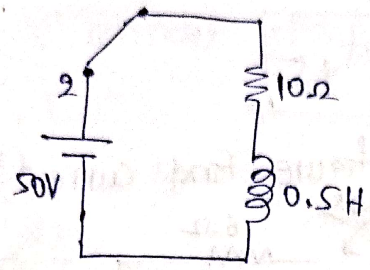
①. Initially at $t = 0^-$, inductor is replaced with open ckt.




$10V = i10$

$\boxed{i = 1}$

②. At $t = 0$.




Apply KVL.

$$50 = i(t)\,10 + 0.5\frac{di(t)}{dt}.$$

divide eqn with 0.5.

$$\frac{di(t)}{dt} + \frac{10}{0.5}\,i(t) = \frac{50}{0.5}$$

$$\frac{di(t)}{dt} + 20\,i(t) = 100 \text{ — ①}.$$

WKT standard 1st order differential eqn is.

$$dx + Px = k \text{ — ②}.$$

$$x = ce^{-Pt} + e^{-Pt}\int k\,e^{Pt}\,dt \text{ — ③}.$$

Compare ① & ②. sub values in —③.

$$i(t) = ce^{-20t} + e^{-20t}\int 100\,e^{20t}\,dt.$$

$$i(t) = ce^{-20t} + e^{-20t}\,.\cancel{100}^{5}.\frac{e^{20t}}{\cancel{20}}$$

$$i(t) = ce^{-20t} + 5\cancel{e^{0}}\,1$$

$$i(t) = ce^{-20t} + 5 \text{ — ④}.$$

To find 'C' consider initial conditions.

at $t = 0^-$, $i = 1$

$$1 = C\cancel{e^{0}} + 5$$

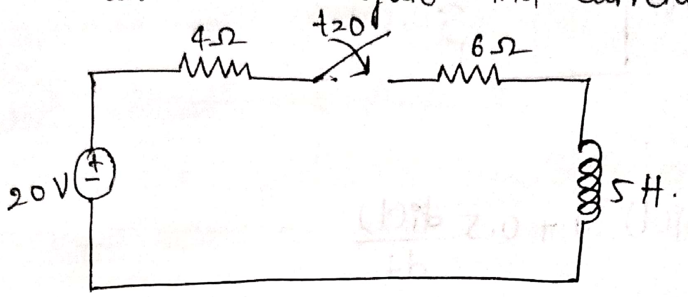
$$C = 1 - 5 \Rightarrow \boxed{C = -4}$$

Sub 'C' in ④.

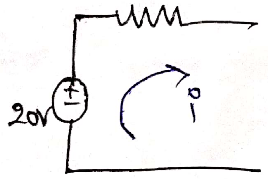
$$\boxed{i(t) = -4e^{-20t} + 5}$$

09/11/2023

②. In the circuit shown in figure. Find current 'i' at t = 3 sec




Sol:- ①. Initial conditions at $t = 0^-$, switch will be opened.

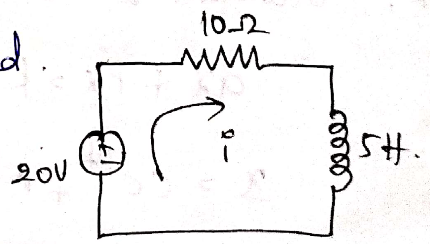



$$20V = 4i$$

$$i = \frac{20}{4} = 5$$

$$\boxed{i = 5A}$$

②. At t = 0, switch will be closed.




Apply KVL

$$20V = i10 + 5\frac{di}{dt}$$

$$\frac{di}{dt} + \frac{10}{5}i = \frac{20}{5}$$

$$\frac{di}{dt} + 2i = 4 \text{ — ①}$$

WKT standard 1st order differential eqn.

$$\frac{dx}{dt} + px = k \text{ — ② where}$$

$$x = Ce^{-pt} + e^{-pt}\int k e^{pt} dt \rightarrow ③$$

Compare ① & ②, substitute in ③.

$i = Ce^{-2t} + e^{-2t}\int 4e^{2t}\,dt$

$i = Ce^{-2t} + e^{-2t}(4)\frac{e^{2t}}{2}$

$i = Ce^{-2t} + 2e^{0}$

$\boxed{i = Ce^{-2t} + 2}$ —— ④

To calculate 'c' consider initial conditions.

→ at $t = 0^-$, $i = 5$

Sub $t$ & $i$ in ④.

$5 = C\cancel{e^{0}}^{1} + 2$

$\boxed{C = 3}$

sub 'c' in ④.

$i = 3e^{-2t} + 2$

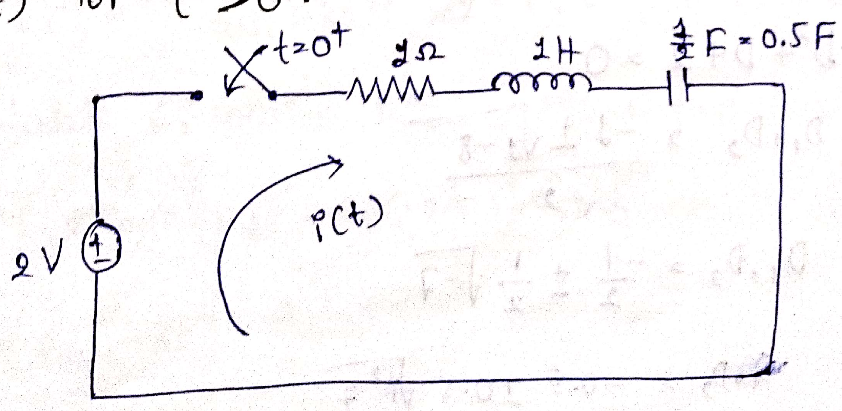
At $t = 3$ sec

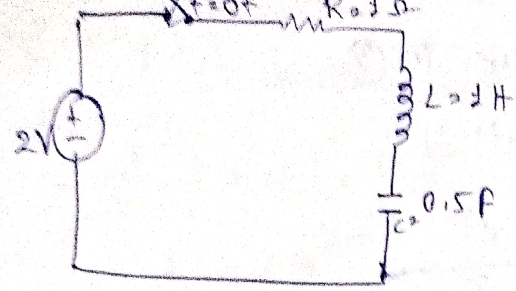
$i = 3e^{-2(3)} + 2$

$i = 3e^{-6} + 2$

$\boxed{i = 2.007 \text{ Amp.}}$

③. $R = 1\Omega$, $L = 1H$, $C = \frac{1}{2}F$ are connected in series across 2 Volts supply. At $t = 0^+$ the switch is closed then find $i(t)$ for $t > 0$.

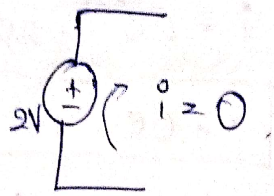






① Initial conditions.

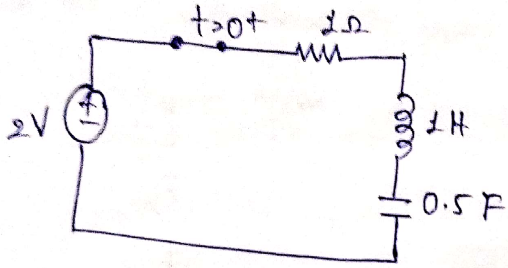
$t=0^-$, switch will be opened.





$$\boxed{i=0}$$

② At $t=0$, switch is closed





Apply KVL.

$$2 = i(1) + 1\frac{di}{dt} + \frac{1}{0.5}\int i\,dt$$

Apply $\frac{d}{dt}$ on Both sides.

$$0 = \frac{di}{dt} + \frac{d^2i}{dt^2} + 2i$$

$$\frac{d^2i}{dt^2} + \frac{di}{dt} + 2i = 0.$$

$$i\left[\frac{d^2}{dt^2} + \frac{d}{dt} + 2\right] = 0$$ Replace $\left[\because \frac{d}{dt} = D\right]$.

$$D^2 + D + 2 = 0.$$

$$D_1, D_2 = \frac{-1 \pm \sqrt{1-8}}{2}$$

$$D_1, D_2 = -\frac{1}{2} \pm \frac{1}{2}\sqrt{-7}$$

$$D_1, D_2 = -0.5 \pm 0.5\sqrt{i^2 7}$$

$$D_1, D_2 = -0.5 \pm 0.5 i\sqrt{7}$$

$$= -0.5 \pm i(0.5)(2.64)$$

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$D_1, D_2 = -0.5 \pm i(1.32)$

$D_1 = -0.5 + i1.32$

$D_2 = -0.5 - i1.32$

As roots are complex conjugate solution 'i' is given by

$$i = e^{k_1(t)}\left[C_1 \cos k_2(t) + C_2 \sin k_2(t)\right]$$

from roots.

$k_1 = -0.5$

$k_2 = 1.32$.

$$i = e^{-0.5t}\left[C_1 \cos(1.32)t + C_2 \sin(1.32)t\right] \quad \text{—(1)}$$

To calculate $C_1$ & $C_2$, consider initial conditions.

At $t = 0^-$, $i = 0$.

$$0 = e^0\left[C_1 \cos(0) + C_2 \sin(0)\right]$$

$$0 = 1[C_1 + 0] \Rightarrow \boxed{C_1 = 0}$$

Substitute '$C_1$' in —(1).

$$i = e^{-0.5t}\left[C_2 \sin(1.32)t\right] \rightarrow (2).$$

perform $\frac{d}{dt}$ $\quad \because \left[\frac{d}{dt}(uv) = uv' + vu'\right]$.

$$\frac{di}{dt} = C_2 e^{-0.5t} \cos(1.32)t \cdot 1.32 + C_2 e^{-0.5t}(0.5) \sin(1.32)t.$$

To calculate '$C_2$' consider initial conditions $t = 0^-$, $i = 0$

$$0 = \left[C_2 e^0 \cos(0)\right]1.32 + \underbrace{C_2(-0.5)e^0 \sin 0}_{0}$$

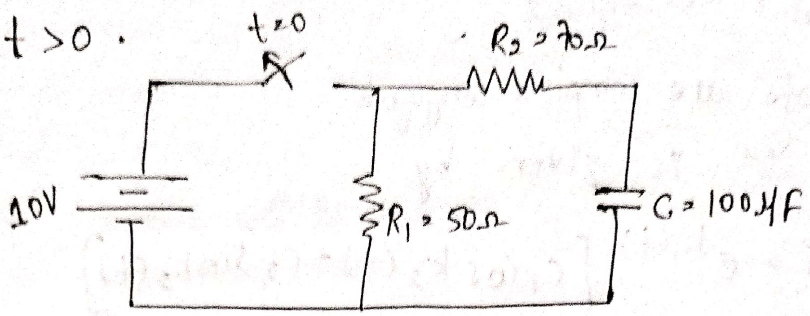
$$0 = C_2(1.32)$$

$$\boxed{C_2 = 0}.$$

Sub $C_1$ & $C_2$ in $eq^n$ —(1)

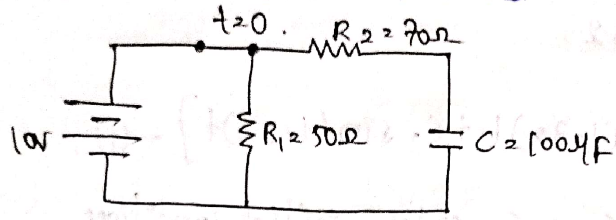
$$\boxed{i = e^{-0.5t}}$$

①. In the network shown in figure, switch 'S' is opened at $t=0$. Find the expression for voltage across the capacitor for $t>0$.




Sol :- ①. Initial conditions.

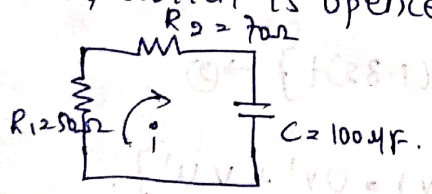
At $t=0^-$ switch is closed.




In this Resistor($R_1$) and Capacitor (c) is connected to the source in parallel. In parallel connection the voltage ~~ac~~ drop across the elements is same i.e. $V_c = 10V$ & $V_{R_1} = 10V$

$V_c = 10v$.

②. At $t=0$, switch is opened




11/11/2023.

Apply KVL

$0 = V_{R_1} + V_{R_2} + V_c$

$= 50i + 70i + V_c$

$= 120i + V_c$ $\quad \left[\because i = c\frac{dv}{dt}\right]$

$0 = 120c\frac{dv_c}{dt} + V_c \Rightarrow 120 \times 100 \times 10^{-6}\frac{dv_c}{dt} + V_c = 0$.

$$\frac{dv_c}{dt} + \frac{V_c}{120 \times 100 \times 10^{-6}} = 0 \quad \text{—①}.$$

WKT.

$\frac{dx}{dt} + Px = k$ — (5). where

$x = ce^{-pt} + e^{-pt} \int k e^{pt} dt$ — (5)

Compare (1) & (5), sub in (3).

$V_c = ce^{-0.012t} + e^{-0.012t} \int 0$

$V_c = ce^{-0.012t}$ — (4)

To calculate 'c' consider initial conditions.

at $t = 0^-$, $V_c = 10V$

$10 = ce^{0}$

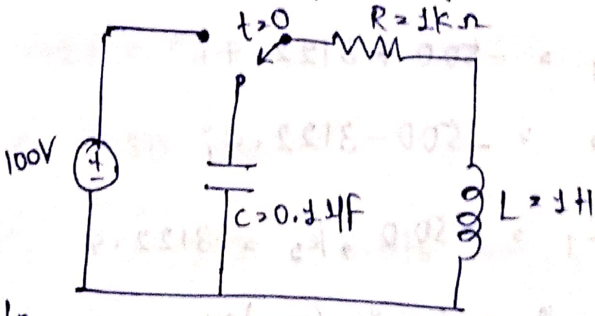
$\boxed{c = 10}$

sub 'c' in eq$^n$ (4).

$$\boxed{V_c = 10e^{-0.012t}}$$

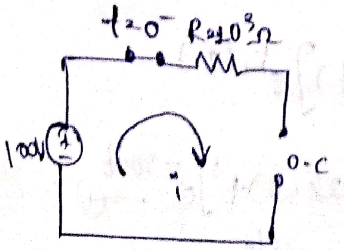
(5). A coil of $R = 1k\Omega$ and $L = 1H$ is connected to a D.C voltage of 100 volts through a change over switch. At $t=0$, the switch connects a capacitor of $C = 0.1\mu F$ in series with the coil excluding voltage. Solve for $i, \frac{di}{dt}, \frac{d^2i}{dt^2}$ at $t = 0^+$.

Sol:-




(1). Initial condition

at $t = 0^-$, switch will be connected to source



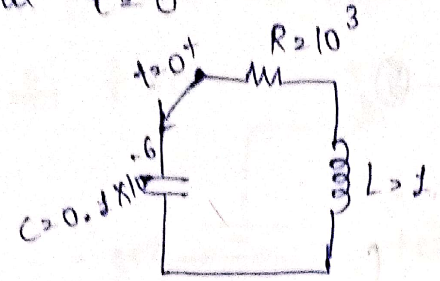

Apply KVL.

$100V = i \times 10^3$

$i = \frac{100}{1000} = 0.1$ Amp.

at $t = 0^-$, $i = 0.1$ Amp.

② at $t = 0^+$




$$0 = i10^3 + \frac{di}{dt} + \frac{1}{0.1\times10^{-6}}\int i\,dt = 0.$$

Apply $\frac{d}{dt}$ on Both sides

$$0 = 10^3\frac{di}{dt} + \frac{d^2i}{dt^2} + \frac{i}{0.1\times10^{-6}}$$

$$\frac{d^2i}{dt^2} + 10^3\frac{di}{dt} + \frac{i}{0.1\times10^{-6}} = 0$$

$$i\left[\frac{d^2}{dt^2} + 10^3\frac{d}{dt} + 10^7\right] = 0.$$

$$D^2 + 10^3 D + 10^7 = 0.$$

$$D_1, D_2 = \frac{-10^3 \pm \sqrt{(10^3)^2 - 4\times10^7}}{2}$$

$$D_1, D_2 = -500 \pm 3122.4i$$

$$D_1 = -500 + 3122.4i$$

$$D_2 = -500 - 3122.4i$$

$$k_1 = -500,\ k_2 = 3122.4$$

Nature of the roots is complex.

solution $i$ for complex roots is.

$$i = [C_1 \cos k_2(t) + C_2 \sin k_2(t)]e^{k_1(t)}$$

$$i = [C_1 \cos(3122.4)t + C_2 \sin(3122.4)t]e^{-500t} \quad (1)$$

To calculate $C_1$ & $C_2$ consider initial conditions.

at $t = 0^-$, $i = 0.1A$.

$$0.1 = [C_1 \cos(0) + C_2 \sin 0] e^{0}$$

$$0.1 = C_1$$

$$\boxed{C_1 = 0.1}$$

Sub $C_1$ in eqn ①.

$$i = [0.1 \cos(3122.4)t + C_2 \sin(3122.4)t] e^{-500t}$$

$$i = 0.1 e^{-500t} \cos(3122.4)t + C_2 e^{-500t} \sin(3122.4)t \quad \text{—②}$$

$\Rightarrow$ find $C_2$ apply $\frac{d}{dt}$ on B.S.

$$\frac{di}{dt} = 0.1 \left[\frac{d}{dt}\left[e^{-500t} \cos(3122.4)t\right]\right] + C_2 \left[\frac{d}{dt}\left[e^{-500t} \sin(3122.4)t\right]\right]$$

Here $u = e^{-500t}$ $\quad V = \cos(3122.4)t$. $\quad \left(\because \frac{d}{dt}(uv) = uv' + vu'\right)$

$u' = e^{-500t} \times (-500)$ $\quad v' = -\sin(3122.4)t \times 3122.4$

$$\frac{di}{dt} = 0.1 \left[e^{-500t} \times (-\sin(3122.4)t) \times 3122.4 + \cos(3122.4)t \times e^{-500t} \times (-500)\right]$$

$$+ C_2 \left[e^{-500t} \times \cos(3122.4)t \times 3122.4 + \sin(3122.4)t \times e^{-500t} \times (-500)\right]$$

Now, consider initial condition at $t = 0^-$, $i = 0.1$ A

$$0 = 0.1 [e^0 \times 0 \times 0 + \cos(0) \times e^0 \times -500] + C_2 [e^0 \times \cos(0) \times 3122.4 + 0]$$

$$0 = 0.1 [-500] + C_2 [3122.4] \quad \because \begin{bmatrix} \cos 0 = 1 \\ \sin 0 = 0 \end{bmatrix}$$

$$0 = -50 + 3122.4\, C_2$$

$$3122.4\, C_2 = 50$$

$$C_2 = \frac{50}{3122.4} = 0.016 \rightarrow \boxed{C_2 = 0.016}$$

Substitute $C_1$ & $C_2$ in eqn ②.

$$i = 0.1 e^{-500t} \cos(3122.4)t + C_2 e^{-500t} \sin(3122.4)t$$

$$i = 0.1 e^{-500t} \cos(3122.4)t + 0.016\, e^{-500t} \sin(3122.4)t.$$

$$\frac{di}{dt} = 0.1 \left[\frac{d}{dt}\left(e^{-500t} \cos(3122.4)t\right]\right] + 0.016 \left[\frac{d}{dt}\left(e^{-500t} \sin(3122.4)t\right)\right]$$

$$\left(\because \frac{d}{dt}(uv) = uv' + vu'\right)$$

$\frac{di}{dt} = 0.2 [e^{-500t} \times \{-\sin(3122.4)t\} \times 3122.4 + \cos(3122.4)t\} \times e^{-500t} \times (-500)]$
$+ 0.016 [e^{-500t} \times \cos(3122.4)t\} \times 3122.4 + \sin(3122.4)t\} \times e^{-500t} \times (-500)]$

$\frac{di}{dt} = [e^{-500t} (-\sin(3122.4)t\} \times 312.24 + \cos(3122.4)t)(e^{-500t})(-100)] +$
$[e^{-500t} \times \cos(3122.4)t\} \times 99.95 + \sin(3122.4)t\} \times e^{-500t} \times$

$\frac{di}{dt} = [0.05 (\cos 3122.4)t\} - 309.25 (\sin(3122.4))] e^{-500t}$

$\frac{di}{dt} = 0.05 [\cos(3122.4)t e^{-500t}] - 309.25 [\sin(3122.4)t e^{-500t}]$

$\frac{d^2i}{dt^2} = 0.05 [\cos(3122.4)t \times e^{-500t} \times -500t + (-\sin(3122.4)t) e^{-500t} \times (3122.4)] +$
$- 309.25 [\sin(3122.4)t e^{-500t} \times (-500) + \cos(3122.4)t e^{-500t} \times 3122.4]$

$\frac{d^2i}{dt^2} = [25 \cos(3122.4)t e^{-500t} - 2841.8 (\sin(3122.4)t e^{-500t}] + [152120 \sin(3122.4)t e^{-500t} + 965958.9 \cos(3122.4)t \times e^{-500t}]$

$\frac{d^2i}{dt^2} = [25 \cos(3122.4)t\} - 2341.8 \sin(3122.4)t\} + 152120 \sin(3122.4)t +$
$965589 \cos(3122.4)t\}] e^{-500t}$

$\frac{d^2i}{dt^2} = [950333.9 \cos(3122.4)t + 149228.9 \sin(3122.4)t] e^{-500t}$

15/11/2023

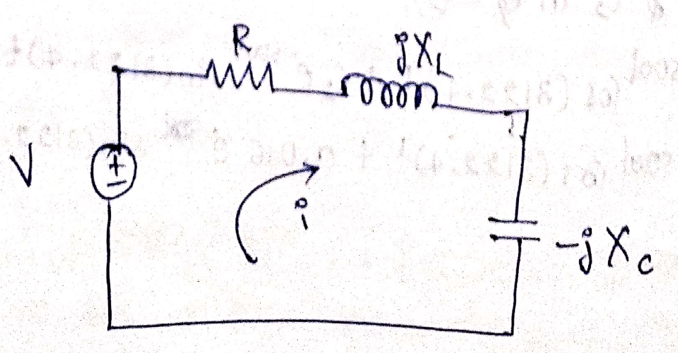
## Resonance

Resonance is a phenomenon at which the net reactance of a circuit is zero. i.e., inductive reactance is equal to capacitive reactance.

⇒ When a circuit is under resonance maximum amount of power can be transmitted.

There exists 2 types of Resonance :- 1) Series Resonance
2) Parallel Resonance

1) Series Resonance

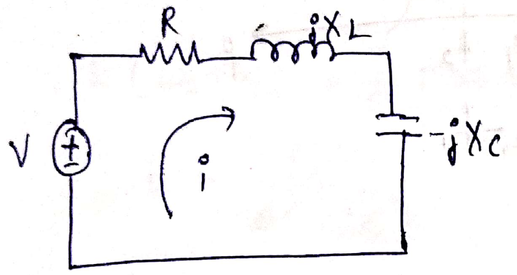


⇒ In a series RLC circuit, under resonance i.e., at $X_L = X_C$ the current flowing through the circuit is Inphase ($\phi = 0$) with applied voltage.

⇒ power factor for series RLC circuit is $\cos\phi = \cos 0 = 1$.

(1) Resonant frequency ($\omega_0$) :- The frequency at which output signal as maximum amplitude is said to be resonant frequency.

→ The Resonant frequency is represented with $f_r / f_0$ (Hz) & $\omega_r / \omega_0$ (bandwidth) (Radian, sec).




Apply KVL

$$V = IR + IjX_L - IjX_C$$

$$V = I[R + jX_L - jX_C]$$

$$V = I[R + j(X_L - X_C)]$$

$V = IZ$ — (1), At resonance

$$X_L = X_C$$

$$\omega L = \frac{1}{\omega C}$$

$$2\pi f_r L = \frac{1}{2\pi f_r C}$$

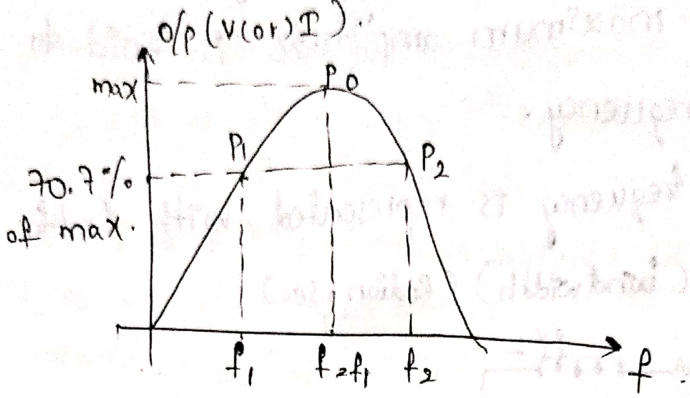
$$f_r^2 = \frac{1}{4\pi^2 LC}$$

$$\boxed{f_r = \frac{1}{2\pi\sqrt{LC}}}$$

Bandwidth :– It is the range of frequencies at which output can become 70.7 % of its maximum value at resonant frequency.

→ It is represented with $BW = f_2 - f_1$.

★★ $f_1$ & $f_2$ are called as half power frequencies.




$BW = f_2 - f_1$

from ①. $V = IZ$

$$I = \frac{V}{Z}.$$

at half power frequency

★★★

$$I = \frac{I_0}{\sqrt{2}}, \quad Z = |Z|$$

$$\frac{I_0}{\sqrt{2}} = \frac{V}{|Z|}$$

$$\frac{\cancel{V}}{R\sqrt{2}} = \frac{\cancel{V}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$R\sqrt{2} = \sqrt{R^2 + (X_L - X_C)^2}$$

Square on B.S.

$$2R^2 = R^2 + (X_L - X_C)^2$$

$$R^2 = (X_L - X_C)^2$$

★ $\pm R = X_L - X_C$

let $-R = X_{L_1} - X_{C_1}$

$$-R = \omega_1 L - \frac{1}{\omega_1 C} \longrightarrow (a)$$

$$+R = X_{L_2} - X_{C_2}$$

$$+R = \omega_2 L - \frac{1}{\omega_2 C} \longrightarrow (b).$$

case-(1).

(b) + (a)

$$\omega_2 L - \frac{1}{\omega_2 C} + \omega_1 L - \frac{1}{\omega_1 C} = \cancel{R - R} \to 0$$

$$L(\omega_2 + \omega_1) - \frac{1}{C}\left[\frac{1}{\omega_2} + \frac{1}{\omega_1}\right] = 0$$

$$L\,\cancel{(\omega_2+\omega_1)} = \frac{1}{C}\left[\frac{\cancel{\omega_1+\omega_2}}{\omega_1\omega_2}\right]$$

$$L = \frac{1}{C(\omega_1\omega_2)}$$

$$\boxed{\omega_1 \omega_2 = \frac{1}{LC}} \longrightarrow (c).$$

Case-(2).

(b) − (a).

$$\omega_2 L - \frac{1}{\omega_2 C} - \omega_1 L + \frac{1}{\omega_1 C} = R - (-R)$$

$$L(\omega_2 - \omega_1) + \frac{1}{C}\left[\frac{1}{\omega_1} - \frac{1}{\omega_2}\right] = 2R.$$

divide eqn with 'L'.

$$(\omega_2 - \omega_1) + \frac{1}{LC}\left[\frac{\omega_2 - \omega_1}{\omega_1\omega_2}\right] = \frac{2R}{L}$$

$$(\omega_2 - \omega_1)\left[1 + \frac{1}{LC}\cdot\frac{1}{\omega_1\omega_2}\right] = \frac{2R}{L}$$

$$(\omega_2 - \omega_1)\left[1 + \frac{1}{LC}\cdot\frac{LC}{1}\right] = \frac{2R}{L} \qquad [\because \text{from } (c)].$$

$$\cancel{2}(\omega_2 - \omega_1) = \frac{\cancel{2}R}{L}$$

$$\omega_2 - \omega_1 = \frac{R}{L}$$

$$2\pi f_2 - 2\pi f_1 = \frac{R}{L}$$

$$2\pi(f_2 - f_1) = \frac{R}{L}$$

$$f_2 - f_1 = \frac{R}{2\pi L}$$

$$BW = \frac{R}{2\pi L}.$$

$$\boxed{BW = f_2 - f_1 = \frac{R}{2\pi L}}$$ for series RLC ckt.

* Half Power frequencies.

$$f_1 = f_r - \frac{R}{4\pi L}$$

$$f_2 = f_r + \frac{R}{4\pi L}$$

Quality factor (or) figure of merit (Q).

Quality factor is defined as the ratio of reactance to resistance.

Case (i) :- $Q = \frac{X_L}{R} = \frac{\omega_L}{R} = \frac{2\pi f_r L}{R}$ —① .

Case (ii) :- $Q = \frac{X_C}{R} = \frac{1}{\omega_C R} = \frac{1}{2\pi f_r C R}$ —② .

Relation between Bandwidth & Quality factor.

WKT, $f_2 - f_1 = \frac{R}{2\pi L}$

divide above eqn with '$f_r$'.

$$\frac{f_2 - f_1}{f_r} = \frac{R}{2\pi L f_r}$$

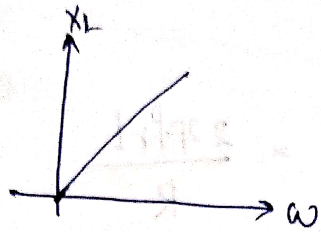
$$\frac{BW}{f_r} = \frac{1}{Q} \quad [\because \text{from } ①]$$

$$\boxed{Q = \frac{f_r}{BW}}$$

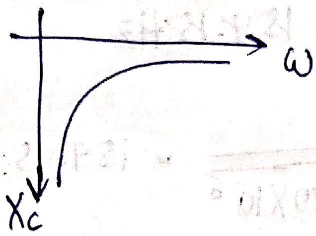
Resonance curves (or) Reactive curves at resonance.

Defⁿ:- The graphical relation of $\omega$ with respect to inductive reactance ($X_L$), Capacitive reactance ($X_C$) and an Impedance ($Z$) is called resonance curves of series RLC circuit.

①. $X_L = \omega L$




②. $X_C = \frac{1}{\omega C}$

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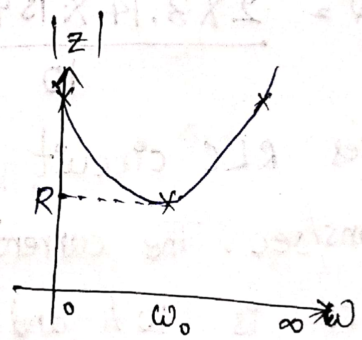
③. $\omega$ with respect to 'Z'

$$Z = R + j(X_L - X_C)$$

$$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$$

$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

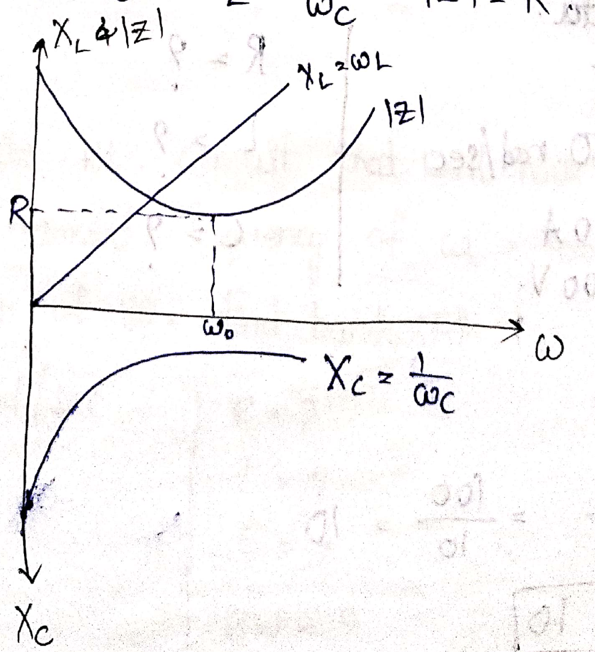



ⓐ. If $\omega = 0 \Rightarrow |Z| = \infty$

ⓑ. If $\omega = \infty \Rightarrow |Z| = \infty$

ⓒ. If $\omega = \omega_0 \Rightarrow X_L = X_C \Rightarrow \omega L = \frac{1}{\omega C} \Rightarrow |Z| = R$

$\Rightarrow X_L$ & $X_C$ & $Z$.



## Problems

① . Determine the quality factor of a coil for series RLC circuit, consisting of $R = 10\,\Omega$, $L = 0.1\,H$ and $C = 10\,\mu F$.

Sol :-
$R = 10\,\Omega$
$L = 0.1\,H$
$C = 10\,\mu F$.

$$Q = \frac{X_L}{R} = \frac{\omega L}{R} = \frac{2\pi f_r L}{R}.$$

$$f_r = \frac{1}{2\pi\sqrt{LC}} = 159.15\,Hz.$$

$$= \frac{1}{2\pi\sqrt{0.1 \times 10 \times 10^{-6}}} = 159.15\,Hz.$$

$$Q = \frac{2 \times 3.14 \times 159.15 \times 0.1}{10} = 9.99 \cong 10$$

②. A series RLC circuit has a quality factor of 5 at 50 radians/sec. The current flowing through the circuit at resonance is 10 A and the supply voltage is 100 volts. find circuit constants.

Sol: Given data.

| | |
|---|---|
| $Q = 5$ | $R = ?$ |
| $\omega_0 = 50$ rad/sec | $L = ?$ |
| $I = 10A$ | $C = ?$ |
| $V = 100\,V$ | |

To find 'R'

$$V = IZ$$

$$Z = \frac{V}{I} = \frac{100}{10} = 10$$

$$\boxed{Z = 10}$$

at resonance $Z = R \Rightarrow \boxed{R = 10\,\Omega}$

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To find 'L'.

WKT

$$Q = \frac{X_L}{R}$$

$$= \frac{\omega L}{R}$$

$$Q = \frac{2\pi f_r L}{R}$$

$$\omega_0 = 50$$

$$2\pi f_r = 50$$

$$f_r = \frac{50}{2\pi} = 7.95\ Hz.$$

$$Q \Rightarrow 5 = \frac{2\pi \times 7.95 \times L}{10} = 1H.$$

To find 'c'

WKT $Q = \frac{X_C}{R}$ (or) $f_r = \frac{1}{2\pi\sqrt{LC}} = 4.004 \times 10^{-4} F$

$$= \frac{1}{\omega_C R}$$

$$7.95 = \frac{1}{2\pi\sqrt{1 \times C}}$$

3) Design a series RLC circuit that will have an impedance of $10\Omega$ at resonant frequency of $\omega_0 = 100$ rad/sec and the quality factor of 80. Find bandwidth.

Sol: $\omega_0 = 100$ rad/sec | $R = ?$

$Q = 80$ | $L = ?$

$Z = 10\Omega$ | $C = ?$

$BW = ?$

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At resonance $Z = R$ i.e. $\boxed{R = 10\Omega.}$

$$Q = \frac{X_L}{R}$$

$$= \frac{\omega L}{R}$$

To find 'L'.

$$Q = \frac{2\pi f_r L}{R}$$

$$\omega_0 = 100$$

$$2\pi f_r = 100 \Rightarrow f_r = \frac{100}{2\pi} = 15.915$$

$$80 = \frac{2\pi \times 15.915 \times L}{10} \Rightarrow L = \frac{800}{2\pi \times 15.915}$$

$$\boxed{L = 8.0002\,H}$$

To find 'c'.

$$Q = \frac{X_c}{R} = \frac{1}{\omega_c R} = \frac{1}{2\pi f_r C R}$$

$1.25 \times 10^{-5}$

$$80 = \frac{1}{2\pi \times 15.915 \times C \times 10}$$

$$C = \frac{1}{2\pi \times 15.91 \times 10 \times 80}$$

$$\boxed{C = 1.25 \times 10^{-5}\,F}$$

To find 'Bandwidth'

WKT.

$$Q = \frac{f_r}{BW} \Rightarrow 80 = \frac{15.91}{BW}$$

$$BW = \frac{15.91}{80} \Rightarrow \boxed{BW = 0.1988}$$

(or)

$$BW = \frac{R}{2\pi L} \Rightarrow \frac{10}{2\pi \times 8} \Rightarrow \boxed{BW = 0.1989}$$

(4). A Series RLC circuit consists of resistance of 25Ω inductance 0.4 H and capacitance of 250 μF. is connected to a supply of 230 Volts, 50 Hz. find total impedance, current, power factor, Voltage across the coil and capacitance.

Sol: Given data,

$R = 25\Omega$

$L = 0.4\,H$

$C = 250\,\mu F$

$V = 230\,V, 50\,Hz.$

$Z = ?$

$I = ?$

Power factor $= \cos\phi = 1$

$V_L = ?$

$V_C = ?$

at resonance

$Z = R = 25\Omega \Rightarrow \boxed{Z = 25}$

WKT. $V = IZ$

$I = \frac{V}{Z} = \frac{230}{25} = 9.2\,Amp. \Rightarrow \boxed{I = 9\,Amp}$

⇒ Power factor for series RLC ckt is '1'.

Voltage across coil

$V_L = I j X_L$

$= j I \omega L$

$V_L = j I (2\pi f_r) L$

$f_r = \frac{1}{2\pi\sqrt{LC}}$

$= \frac{1}{2\pi\sqrt{0.4 \times 250 \times 10^{-6}}}$

$f_r = 15.9\,Hz.$

$\therefore V_L = j(9)(2\pi \times 15.9)(0.4)$

$\boxed{V_L = j\,367.8\,V}$

Voltage across capacitor.

$V_C = I(-jX_C)$

$= -jI\frac{1}{\omega C}$

$= -j\frac{I}{2\pi f_r C}$

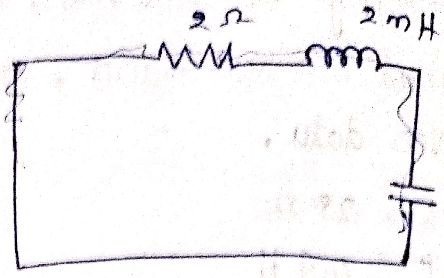
$V_C = -j\frac{9}{2\pi \times 15.9 \times 250 \times 10^{-6}}$

$= -j\,368.1\,V$

$\therefore \boxed{V_C = -368.1 j\,V}$

17/4/2

⑧ In a series RLC circuit $R = 2\Omega$, $L = 2\,mH$, $C = 10\mu F$
find resonant frequency, half power frequencies, bandwidth
and Quality factor.




Sol: $R = 2\Omega$

$L = 2\,mH$

$C = 10\mu F$.

Resonant frequency:

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{2\times10^{-3}\times10\times10^{-6}}} = 1125.39\,Hz.$$

Half power frequencies:

$$f_1 = f_r - \frac{R}{4\pi L} = \cancel{1125}\ 1125.39 - \frac{2}{4\pi\times2\times10^{-3}} = 1045.$$

$$f_2 = f_r + \frac{R}{4\pi L} = 1125.3 + \frac{2}{4\pi\times2\times10^{-3}} = 1204.87\,Hz.$$

Bandwidth

$$BW = \frac{R}{2\pi L}$$

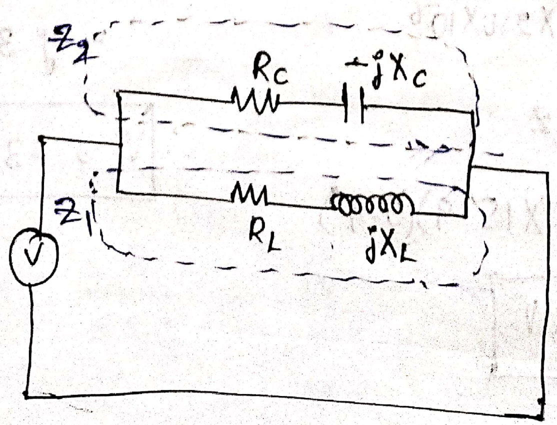
$$BW = f_2 - f_1 = 159.17\,Hz.$$

Quality factor:

$$Q = \frac{f_r}{BW} = \frac{1125.39}{159.17} = 7.07.$$

## Parallel Resonance

In a parallel RLC circuit, under the resonance condition ($X_L = X_C$), two branch currents are equal in magnitude but $180^\circ$ out of phase with eachother.



$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2}$$

$$Y = Y_1 + Y_2$$

$$Y = \frac{1}{R_L + jX_L} + \frac{1}{R_C - jX_C}$$

$$Y = \frac{1}{R_L + j\omega L} + \frac{1}{R_C - \frac{j}{\omega C}}$$

$$Y = \frac{1}{R_L + j\omega L} \times \frac{R_L - j\omega L}{R_L - j\omega L} + \frac{1}{R_C - \frac{j}{\omega C}} \times \frac{R_C + j/\omega C}{R_C + j/\omega C}$$

$$Y = \frac{R_L - j\omega L}{R_L^2 + \omega^2 L^2} + \frac{R_C + \frac{j}{\omega C}}{R_C^2 + \frac{1}{\omega^2 C^2}}$$

separate real & imaginary terms

$$Y = \frac{R_L}{R_L^2 + \omega^2 L^2} - \frac{j\omega L}{R_L^2 + \omega^2 L^2} + \frac{R_C}{R_C^2 + \frac{1}{\omega^2 C^2}} + \frac{j}{\omega C\left(R_C^2 + \frac{1}{\omega^2 C^2}\right)}$$

$$Y = \frac{R_L}{R_L^2 + \omega^2 L^2} + \frac{R_C}{R_C^2 + \frac{1}{\omega^2 C^2}} + j\left[\frac{1}{\omega C\left(R_C^2 + \frac{1}{\omega^2 C^2}\right)} - \frac{\omega L}{R_L^2 + \omega^2 L^2}\right]$$

at resonance ($X_L = X_C$), imaginary part $= 0$.

$$\frac{1}{\omega C\left(R_C^2 + \frac{1}{\omega^2 C^2}\right)} - \frac{\omega L}{R_L^2 + \omega^2 L^2} = 0$$

$$\frac{1}{\omega C\left(R_C^2 + \frac{1}{\omega^2 C^2}\right)} = \frac{\omega L}{R_L^2 + \omega^2 L^2}$$

$$R_L^2 + \omega^2 L^2 = \omega^2 LC\left[\frac{R_C^2 \omega^2 C^2 + 1}{\omega^2 C^2}\right]$$

$$(R_L^2 + \omega^2 L^2)\,\omega^2 C^2 = \omega^2 LC\,(R_C^2 \omega^2 C^2 + 1)$$

$$(R_L^2 + \omega^2 L^2)\,C = (R_C^2 \omega^2 C^2 + 1)\,L$$

$$R_L^2 C + \omega^2 L^2 C = R_C^2 \omega^2 C^2 L + L$$

$$\omega^2 L^2 C - R_C^2 \omega^2 C^2 L = L - R_L^2 C$$

$$\omega^2 LC\,(L - R_C^2 C) = L - R_L^2 C$$

$$\omega^2 LC = \frac{L - R_L^2 C}{L - R_C^2 C}$$

case (i):- $\boxed{\text{if } R_L = R_C = R}$

Case(i) :- If $R_L = R_C = R$

$$\omega^2 LC = \frac{\cancel{L-RC}}{L-RC}$$

$$\omega^2 LC = 1$$

$$\omega^2 = \frac{1}{LC} \Rightarrow \omega = \frac{1}{\sqrt{LC}}$$

$$2\pi f_r = \frac{1}{\sqrt{LC}} \rightarrow \boxed{f_r = \frac{1}{2\pi\sqrt{LC}}}$$

18/11/2023

Case(ii) :- If $R_L \neq R_C$

$$\omega^2 LC = \frac{L - R_L^2 C}{L - R_L^2 C}$$

$$\omega^2 = \frac{1}{LC}\left[\frac{L - R_L^2 C}{L - R_C^2 C}\right]$$

$$\omega = \frac{1}{\sqrt{LC}}\left[\frac{L - R_L^2 C}{L - R_C^2 C}\right]$$

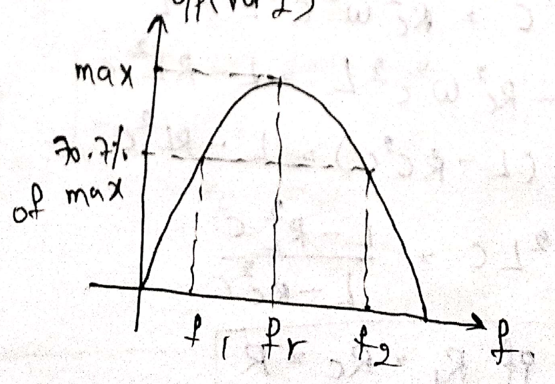
$$= \frac{1}{2\pi\sqrt{LC}}\left[\frac{L - R_L^2 C}{L - R_C^2 C}\right]$$

$$\boxed{f_r = \frac{1}{2\pi\sqrt{LC}}\left[\frac{L - R_L^2 C}{L - R_C^2 C}\right]}$$

**Bandwidth** :- It is the range of frequencies at which output can become 70.7% of its maximum value at resonant frequency.

⇒ It is represented with $BW = f_2 - f_1$

$f_1$ & $f_2$ are called as half power frequencies.



for parallel RLC circuit Admittance is given by

$$Y = \frac{1}{R} + j\omega C - \frac{j}{\omega L}$$

$$Y = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

at resonance

Let $-\frac{1}{R} = \omega_1 C - \frac{1}{\omega_1 L} \rightarrow (a)$

$+\frac{1}{R} = \omega_2 C - \frac{1}{\omega_2 L} \rightarrow (b)$

case (i) (b) + (a)

$$\omega_2 C - \frac{1}{\omega_2 L} + \omega_1 C - \frac{1}{\omega_1 L} = \cancel{\frac{1}{R} - \frac{1}{R}} \rightarrow 0$$

$$c(\omega_1 + \omega_2) - \frac{1}{L}\left(\frac{1}{\omega_1} + \frac{1}{\omega_2}\right) = 0$$

$$c(\cancel{\omega_1 + \omega_2}) = \frac{1}{L}\left(\frac{\cancel{\omega_2 + \omega_1}}{\omega_1 \omega_2}\right)$$

$$C = \frac{1}{L\omega_1\omega_2} \Rightarrow \boxed{\omega_1 \omega_2 = \frac{1}{LC}}$$

case (ii) (b) − (a)

$$\omega_2 C - \frac{1}{\omega_2 L} - \omega_1 C + \frac{1}{\omega_1 L} = \frac{1}{R} + \frac{1}{R}$$

$$c(\omega_2 - \omega_1) + \frac{1}{L}\left(\frac{1}{\omega_1} - \frac{1}{\omega_2}\right) = \frac{2}{R}$$

% eqn with 'c'

$$(\omega_2 - \omega_1) + \frac{1}{LC}\left(\frac{\omega_2 - \omega_1}{\omega_1 \omega_2}\right) = \frac{2}{RC}$$

$$(\omega_2 - \omega_1)\left[1 + \frac{1}{LC} \cdot \frac{1}{\omega_1 \omega_2}\right] = \frac{2}{RC}$$

$$(\omega_2 - \omega_1)\left[1 + \frac{1}{LC} \cdot \frac{1}{1/LC}\right] = \frac{2}{RC}$$

$$2(\omega_2 - \omega_1) = \frac{2}{RC}$$

$$\omega_2 - \omega_1 = \frac{1}{RC}$$

$$\boxed{BW = 1/RC \text{ radian/sec}}$$

$$2\pi f_2 - 2\pi f_1 = \frac{1}{RC}$$

$$f_2 - f_1 = \frac{1}{2\pi RC}$$

$$\boxed{BW = \frac{1}{2\pi RC} \ Hz.}$$

Quality factor :- $Q = \frac{f_r}{BW}$

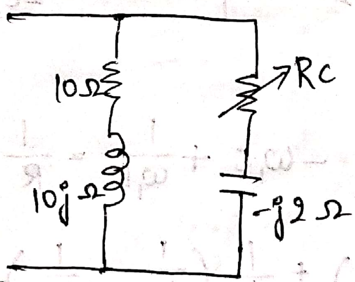
$Q = \frac{f_r}{1/RC}$ | $Q = \frac{f_r}{1/2\pi RC}$

$\boxed{Q = f_r RC}$ rad/sec | $\boxed{Q = f_r 2\pi RC}$ Hz.

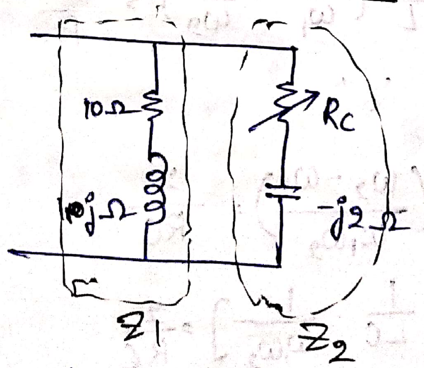
These quality factor is w.r.t. parallel RLC ckt.

Problems

① Calculate the value of RC in the circuit shown figure to yield the resonance.




Sol :-




$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2}$$

$$Y = Y_1 + Y_2$$

$$Y = \frac{1}{10+10j} + \frac{1}{RC-2j}$$

$$Y = \frac{1}{10+10j} + \frac{1}{RC-2j}$$

$$Y = \frac{1}{10+10j} \times \frac{10-10j}{10-10j} + \frac{1}{RC-2j} \times \frac{RC+2j}{RC+2j}$$

$$= \frac{10-10j}{100+100} + \frac{RC+2j}{RC^2+4}.$$

separate real & imaginary terms.

$$Y = \frac{10}{200} - \frac{10j}{200} + \frac{RC}{RC^2+4} + \frac{2j}{RC^2+4}$$

$$Y = \frac{1}{20} + \frac{RC}{RC^2+4} + j\left[\frac{2}{RC^2+4} - \frac{10}{200}\right]$$

at Resonance, imaginary part is zero (parallel RLC).

$$\frac{2}{RC^2+4} - \frac{1}{20} = 0$$

$$\frac{2}{RC^2+4} = \frac{1}{20}$$

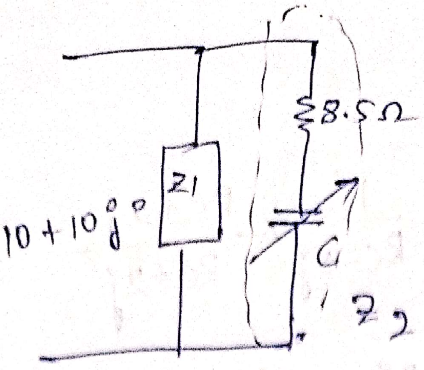
$$RC^2 + 4 = 40 \Rightarrow RC^2 = 40-4 = 36$$

$$RC^2 = 36$$

$$RC = \sqrt{36}$$

$$RC = \pm 6 \Rightarrow \boxed{\begin{array}{l} RC = 6\,\Omega \\ RC = -6\,\Omega \end{array}}$$

②. An impedance $Z_1 = 10 + j10\,\Omega$ is connected in parallel with another impedance of resistance $8.5\,\Omega$ and a variable capacitance connected in series. Find 'C' such that circuit is in resonance at 5kHz.




$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2}$$

$$Y = Y_1 + Y_2$$

$$= \frac{1}{10+10j} + \frac{1}{8.5-jX_C}$$

$$= \frac{1}{10+10j} \times \frac{10-10j}{10-10j} + \frac{1}{8.5-jX_C} \times \frac{8.5+jX_C}{8.5+jX_C}$$

$$= \frac{10-10j}{100+100} + \frac{8.5+jX_C}{72.25+X_C^2}$$

$$= \frac{10}{100+100} - \frac{10j}{100+100} + \frac{8.5}{72.25+X_C^2} + \frac{jX_C}{72.25+X_C^2}$$

$$= \frac{10}{200} - \frac{10j}{200} + \frac{8.5}{72.25+X_C^2} + \frac{jX_C}{72.25+X_C^2}$$

$$= \frac{1}{20} - \frac{10j}{200} + \frac{8.5}{72.25+X_C^2} + \frac{jX_C}{72.25+X_C^2}$$

$$= \frac{1}{20} + \frac{8.5}{72.25+X_C^2} + j\left[\frac{X_C}{72.25+X_C^2} - \frac{10}{200}\right]$$

$$\frac{X_C}{72.25+X_C^2} - \frac{1}{20} = 0$$

$$\frac{X_C}{72.25+X_C^2} = \frac{1}{20}$$

$$20X_C = 72.25 + X_C^2$$

$$X_C^2 - 20X_C + 72.25 = 0$$

$ax^2+bx+c=0$

$$X_c = \frac{20 \pm \sqrt{111}}{2}$$

Case (i)

$X_c = 15.267$

$\frac{1}{\omega C} = 15.267$

$\frac{1}{2\pi f C} = 15.267$

$\frac{1}{C} = 15.267 \times 2\pi \times 5 \times 10^3$ $= 2.04 \times 10^{-6}$

$\frac{1}{C} = 152670\pi$

$C = 2.0849 \times 10^{-6}$

$C = 2.084\,\mu F.$

Case (ii) $= 6.72 \times 10^{-6} F$

$X_c = 4.732.$

$\frac{1}{\omega C} = 4.732$

$\frac{1}{2\pi f C} = 4.732$

$\frac{1}{C} = 4.732 \times 2\pi \times 5 \times 10^3$

$\frac{1}{C} = 47320\pi$

$C = 6.72 \times 10^{-6}$

$C = 6.72\,\mu F$

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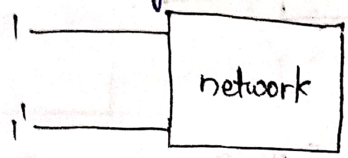
UNIT-3 

# Two-port Network parameters

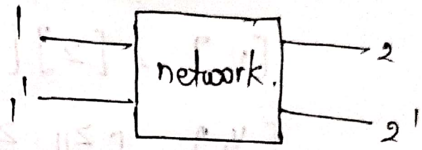
A network with one pair of terminals (or) with one port is said to one-port network.

→ A network with two pairs of terminals (or) with two port is said to Two-port network.

→ A network is represented with a square box in two port network analysis.



ⓐ. one-port n/w

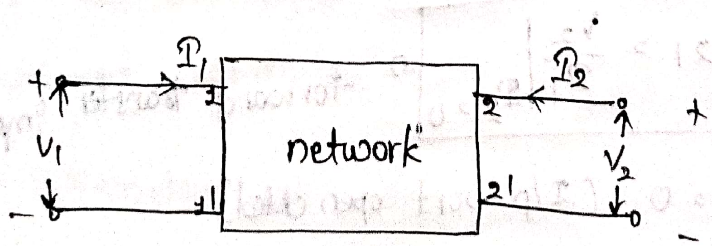


ⓑ. two-port n/w.

→ To analyze a two-port network, following parameters are used.

1) Z parameters. (Impedance parameters).
2) Y parameters (Admittance parameters).
3) h-parameters (hybrid parameters).
4) Inverse of h-parameters (or) g parameters.
5) ABCD parameters (or) Transmission parameters.

1) Z-parameters (or) Impedance parameters.




Input & output voltages $V_1$ and $V_2$ can be represented in the form of currents $I_1$ and $I_2$ with the help of Z-matrix.

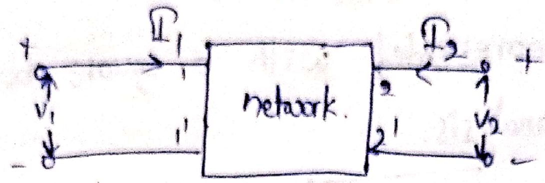
i.e, $[V] = [Z][I]$ → ①.

$[Z] \rightarrow$ Impedance matrix (or) Z-matrix.

$[V] = [Z][I]$

⇒ from the above $eq^n$ it can observed that $V_1$ & $V_2$ are dependent parameters whereas $I_1$ & $I_2$ are Independent parameters with respect to Z - parameters.

★ ⇒ Z parameters are also called as 'open circuited' parameters




$[V] = [Z][I]$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{— (1)}$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{— (2)}$$

Case (i) If $I_2 = 0$ (o/p port open ckted).

from $eq^n$ — (1) ⇒ $V_1 = Z_{11} I_1$

$$\boxed{Z_{11} = \frac{V_1}{I_1}\Big|_{I_2 = 0}} \ \Omega$$ input diving point impedance.

from $eq^n$ — (2) ⇒ $V_2 = Z_{21} I_1$

$$\boxed{Z_{21} = \frac{V_2}{I_1}\Big|_{I_2 = 0}} \ \Omega$$ forward transfer impedance.

Case (ii) If $I_1 = 0$ (I/p port open ckted).

from $eq^n$ — (1). $V_1 = Z_{12} I_2$

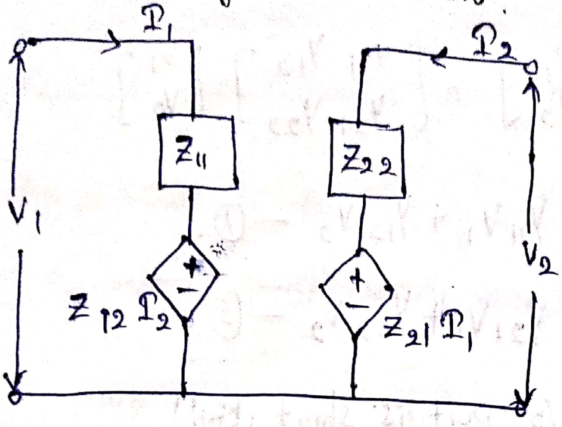
$$\boxed{Z_{12} = \frac{V_1}{I_2}\Big|_{I_1 = 0}} \ \Omega$$

reverse transfer impedance.

from eqⁿ ②. $V_2 = Z_{22} I_2$

$$\boxed{Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}} \Omega$$ o/p driving point impedance.

Equivalent network of a two-port network with respective to Z-parameters is shown below.




Y-parameters (or) Admittance parameters.


Input & output currents $I_1, I_2$ can be represented in the form of voltages $V_1, V_2$. We the help of Y-matrix, i.e.

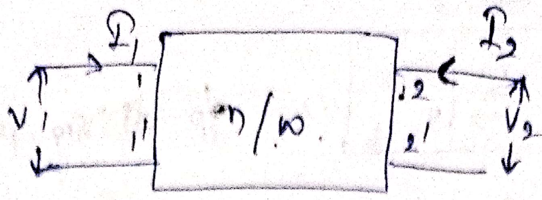
$$[I] = [Y][V]$$

Y → Y matrix (or) admittance matrix.

$\begin{Bmatrix} I_1 \\ I_2 \end{Bmatrix}$

from the above eqⁿ it can be observed that $I_1, I_2$ are dependent parameters where as $V_1, V_2$ are Independent parameters with respect to Y-parameters.

⇒ Y-parameters are also called as "short circuited" parameters.




$$[I] = [Y][V]$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \quad \text{—①}.$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 \quad \text{—②}.$$

Case (i) if $V_2 = 0$ (o/p port is short ckted).

from eqⁿ ①. $I_1 = Y_{11}V_1$

$\boxed{Y_{11} = \frac{I_1}{V_1}}$ ℧ i/p driving point admittance

from eqⁿ ②. $I_2 = Y_{21}V_1$

$\boxed{Y_{21} = \frac{I_2}{V_1}}$ ℧ forward transfer admittance

Case (ii) If $V_1 = 0$ (i/p port is short ckted)

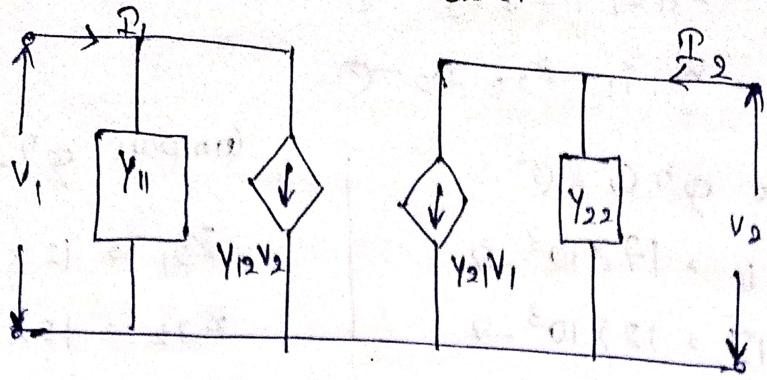
from eqⁿ ①. $I_1 = Y_{12}I_2$

$\boxed{Y_{12} = \frac{I_1}{I_2}}$ ℧ Reverse transfer admittance

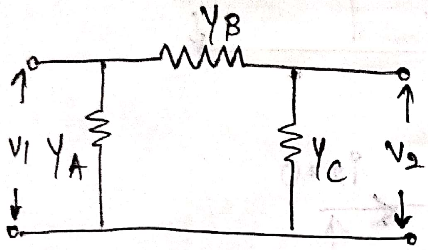
from eqⁿ ②. $I_2 = Y_{22}V_2$

$\boxed{Y_{22} = \frac{I_2}{V_2}}$ ℧ o/p driving point admittance.

⇒ equivalent n/w of a two-port n/w with respect to Y-parameters is shown below.




⇒ If the circuit to be analyzed is in π- structure then its equivalent circuit with respect to Y-parameters is as shown.




$$\left.\begin{aligned} Y_{11} &= Y_A + Y_B \\ Y_{12} = Y_{21} &= -Y_B \\ Y_{22} &= Y_B + Y_C \end{aligned}\right\}$$

where

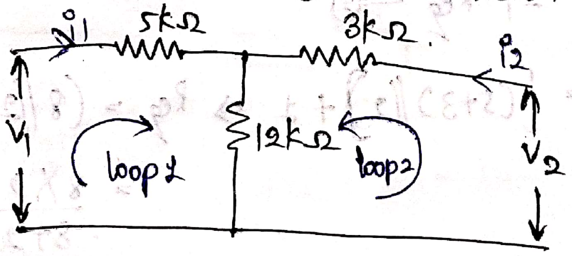
$Y_A = \frac{1}{R_A}$

$Y_B = 1/R_B$

$Y_C = 1/R_C$.

<u>Problems</u> 21/11/2023

1) find z- parameters for the network shown in figure.




Sol: Apply KVL to loop-1 :-

$V_1 = 5\times10^3 i_1 + 12\times10^3(i_1+i_2)$.

$= 5\times10^3 i_1 + 12\times10^3 i_1 + 12\times10^3 i_2$

$V_1 = 17\times10^3 i_1 + 12\times10^3 i_2$ —— ①

Apply KVL to loop-2 :-

$V_2 = 3\times10^3 i_2 + 12\times10^3(i_1+i_2)$

$= 3\times10^3 i_2 + 12\times10^3 i_2 + 12\times10^3 i_1$

$V_2 = 12\times10^3 i_1 + 15\times10^3 i_2$ —— ②

WKT for z-parameters.

$V_1 = Z_{11} I_1 + Z_{12} I_2$ —①

$V_2 = Z_{21} i_1 + Z_{22} I_2$ —②.

Compare eqn ① & ③.

$Z_{11} = 17 \times 10^3 \Omega$

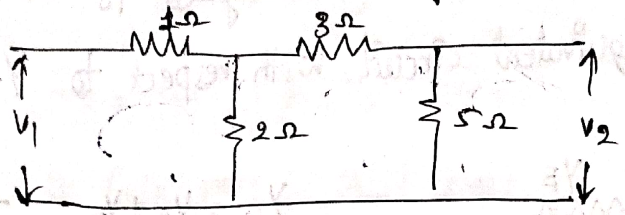
$Z_{12} = 12 \times 10^3 \Omega$.

Compare eqn – ② & ④.

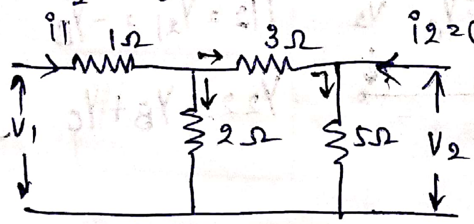
$Z_{21} = 12 \times 10^3 \Omega$

$Z_{22} = 15 \times 10^3 \Omega$ //.

② find z-parameters for following network.




Sol :- Case (i) :- let $i_2 = 0$.




$V_2 = I_{5\Omega} \times 5$ —①

$V_1 = I_1 . R_{eq}$ —②

$R_{eq} = [(5+3) \| 2] + 1 \Rightarrow R_{eq} = (8 \| 2) + 1$

$= \frac{8 \times 2}{8+2} + 1 = \frac{16+1}{10}$

$R_{eq} = \frac{16}{10} + 1 = 2.6 \Omega$.

Sub in eqn ②.

$V_1 = I_1 2.6$

$\frac{V_1}{I_1} = 2.6$

$\boxed{Z_{11} = 2.6}$

$I_{5\Omega} = I_1 \times \frac{2}{2+3+5}$ (∵ Current divider rule)

$I_{5\Omega} = I_1 \times \frac{\cancel{2}}{\cancel{10}_5}$

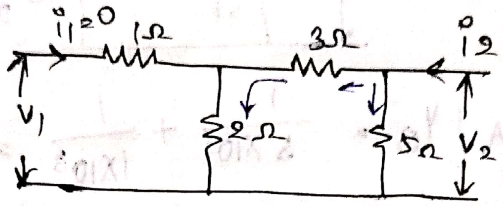
$I_{5\Omega} = \frac{I_1}{5}$

sub in eqⁿ-(1).

$V_2 = \frac{I_1}{\cancel{5}} \times \cancel{5}$

$\frac{V_2}{I_1} = 1 \Rightarrow \boxed{Z_{21} = 1\Omega}$

Case (ii) let $i_1 = 0$.




$V_1 = I_{2\Omega} \times 2$ — (1)

$V_2 = i_2 R_{eqn}$ — (2)

$R_{eqn} = (2+3)||5 = 5||5 = \frac{5\times5}{5+5} = \frac{25}{10} = 2.5$

sub in eqⁿ-(2)

$V_2 = i_2(2.5)$

$\frac{V_2}{i_2} = 2.5 \Rightarrow \boxed{Z_{22} = 2.5\Omega}$

$I_{2\Omega} = I_2 \times \frac{5}{2+3+5}$

$= I_2 \cdot \frac{\cancel{5}^1}{\cancel{10}_2}$

$\Rightarrow I_{2\Omega} = \frac{I_2}{2}$

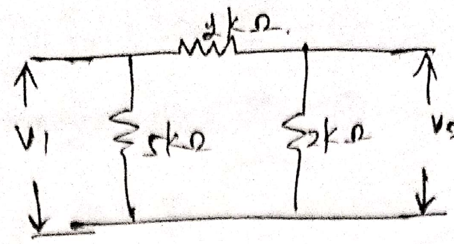
sub in eqⁿ-(1).

$V_1 = \frac{I_2}{\cancel{2}} \cdot \cancel{2}$

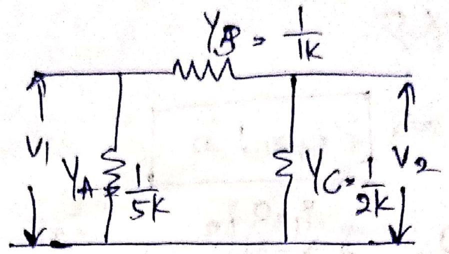
$\frac{V_1}{I_2} = 1$

∴ $\boxed{Z_{12} = 1\Omega}$

③. Find Y-parameters for the circuit shown below.




Sol:- The equivalent Y-parameter circuit for π structure is given by.

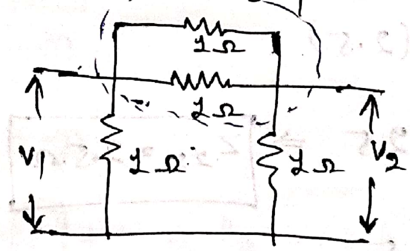



$$Y_{11} = Y_A + Y_B = \frac{1}{5 \times 10^3} + \frac{1}{1 \times 10^3} = 1.2 \times 10^{-3}\ \mho \text{ (or) } 1.2\,m\mho$$

$$Y_{12} = Y_{21} = -Y_B = -\frac{1}{10^3} \text{ or } -\frac{1}{10^3} = 10^{-3}\ \mho = -1\,m\mho$$

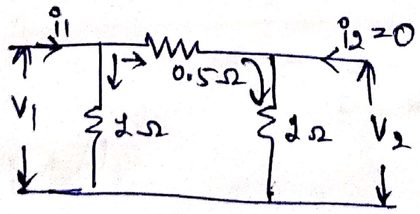
$$Y_{22} = Y_B + Y_C = \frac{1}{10^3} + \frac{1}{2 \times 10^3} = 1.5 \times 10^{-3}\ \mho \text{ (or) } 1.5\,m\mho$$

④. Find Impedance and admittance parameters for the circuit shown below.




Sol:- $R_{eq} = 1 \| 1 = \frac{1}{2} = 0.5$.

Case (i) $i_2 = 0$.




$$V_2 = I_{1\Omega} \times 1 \quad \text{—①.}$$

$$V_1 = I_1 \cdot R_{eq} \quad \text{—②}$$

$$R_{eq} = (0.5+1) \| 1 = 1.5 \| 1 = \frac{1.5 \times 1}{1.5+1}$$

$$= \frac{1.5}{2.5} = 0.6\,\Omega$$

sub in eq – ②.

$V_1 = I_1 \cdot 0.6$

$\frac{V_1}{I_1} = 0.6$

$\boxed{Z_{11} = 0.6}\ \Omega$

$I_{1\Omega} = I_1 \times \frac{1}{1+1+0.5}$

$= I_1 \times \frac{1}{2.5}$

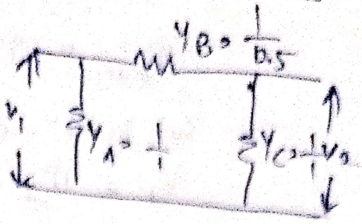
$I_{1\Omega} = \frac{I_1}{2.5}$

Sub in eqn – ①.

$V_2 = \frac{I_1}{2.5} \times 1.$

$\frac{V_2}{I_1} = \frac{1}{2.5} \Rightarrow \boxed{Z_{21} = 0.4}\ \Omega$

Y- parameters

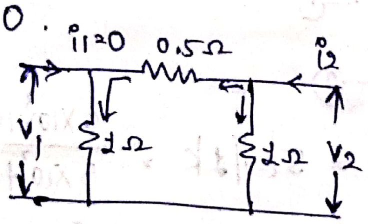



$Y_{11} = Y_A + Y_B = \frac{1}{1} + \frac{1}{0.5} = 3\mho$

$Y_{12} = Y_{21} = -Y_B = -\frac{1}{0.5} = -2\mho$

$Y_{22} = Y_B + Y_C = \frac{1}{0.5} + \frac{1}{1} = 3\mho$

case (ii) :- $i_1 = 0$.




$\Rightarrow V_1 = I_{1\Omega} \times 1$ – ①

$V_2 = i_2 R_{eqn}$ – ②.

$R_{eqn} = (1+0.5) \| 1 = 1.5 \| 1 = \frac{1.5 \times 1}{1.5+1} = 0.6\ \Omega$.

Sub in eqn – ②.

$V_2 = i_2 \cdot 0.6$

$\frac{V_2}{I_2} = 0.6 \Rightarrow \boxed{Z_{22} = 0.6}\ \Omega$

$\Rightarrow I_{1\Omega} = I_2 \times \frac{1}{1+1+0.5} \Rightarrow I_2 \times \frac{1}{2.5}$

sub eqn – ①.

$V_1 = I_2 \cdot \frac{1}{2.5} \times 1 \Rightarrow \frac{V_1}{I_2} = \frac{1}{2.5} = 0.4\ \boxed{Z_{12} = 0.4}\ \Omega$

$$I_{1\Omega} = I_2 \times \frac{1}{1+1+0.5}$$

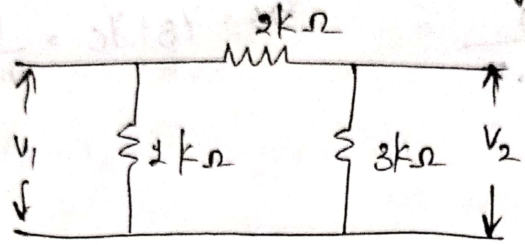
$$I_{1\Omega} = I_2 \times \frac{1}{2.5}$$

Sub in eq$^n$ — ①

$$V_1 = I_2 \cdot \frac{1}{2.5} \times 1$$

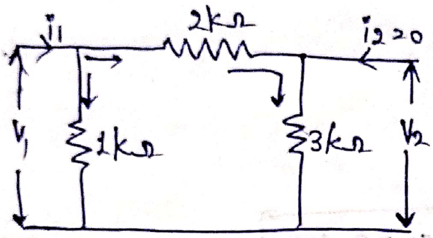
$$\frac{V_1}{I_2} = \frac{1}{2.5} = 0.4 \Rightarrow \boxed{Z_{12} = 0.4}\ \Omega$$

⑤. Find the z-parameters for the given network circuit.




Sol: Case (i)

let $i_2 = 0$




$$V_2 = I_{3k\Omega} \times 3k \text{ — ①}$$

$$V_1 = I_1 \cdot R_{eq} \text{ — ②}$$

$$R_{eq} = (2k+3k) \| 1k = 5k \| 1k = \frac{5\times10^3 \times 10^3}{5\times10^3 + 10^3} = 833.33$$

$$= \frac{5}{6}\times 10^3 = \frac{5}{6}\,k\Omega.$$

Sub in eq$^n$ — ②

$$V_1 = I_1 \cdot 833.33$$

$$\frac{V_1}{I_1} = 833.33 \Rightarrow \boxed{Z_{11} = 833.33}\ \Omega \text{ (or) } \boxed{Z_{11} = \frac{5}{6}\,k\Omega}$$

$$I_{3k\Omega} = I_1 \times \frac{1\times10^3}{1k+2k+3k} = I_1 \times 0.16667 \quad \frac{I_1}{6}.$$

Sub in eq — ①.

$$V_2 = \frac{I_1}{6} \times 3\times10^3$$

$$\frac{V_2}{I_1} = \frac{1}{\cancel{6}_2} \times \cancel{3}\times10^3 = 500 \quad \frac{10^3}{2} = 0.5\,k\Omega$$

$$\boxed{Z_{21} = 500}\ \Omega. \quad \boxed{Z_{21} = 0.5\,k\Omega}$$

case (ii) $i_1 = 0$

$V_2 = I_2 \cdot R_{eq}$ — ①

$V_1 = I_{1k\Omega} \times 1k$ — ②

$R_{eq} = (1\times10^3 + 2\times10^3) \parallel 3\times10^3$

$= \frac{3\times10^3 \times 3\times10^3}{3\times10^3 + 3\times10^3} = \frac{9\times10^3}{6\times10^3} = \frac{3}{2}\times10^3$

$V_2 = I_2 \times \frac{3}{2}\times10^3$

$\frac{V_2}{I_2} = 1.5\times10^3 \Rightarrow \boxed{Z_{22} = 1.5k\Omega}$

$V_1 = I_{1k\Omega} \times 1k$

$I_{1k\Omega} = I_2 \times \frac{3\times10^3}{1\times10^3 + 2\times10^3 + 3\times10^3} = I_2 \frac{3\times10^3}{6\times10^3}$

$I_{1k\Omega} = \frac{I_2}{2}$

$V_1 = 1\times10^3 \times \frac{I_2}{2}$

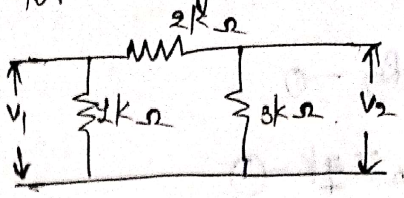
$V_1 = \frac{I_2}{2} \times 1k \Rightarrow V_1 = \frac{I_2}{2} \times 1\times10^3$

$\frac{V_1}{I_2} = \frac{10^3}{2}$

$\boxed{Z_{12} = 0.5\times10^3 \text{ (or) } 0.5k\Omega}$ //

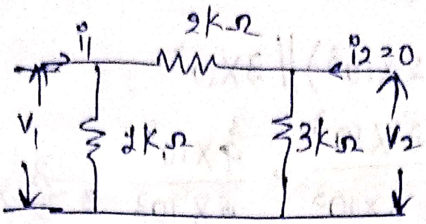
~~$Z_{12} = 0.5\times10$~~

(3). Find the Z-parameters for the given circuit.




Sol:

Case (i) let $i_2 = 0$.




$V_2 = I_{3k\Omega} \times 3k$ — (1)

$V_1 = I_1 R_{eq}$ — (2).

$R_{eq} = (2\times10^3 + 3\times10^3) \| 1\times10^3 = 5\times10^3 \| 1\times10^3 = \frac{5\times10^3 \times 10^3}{5\times10^3 + 10^3} = \frac{5}{6}\times10^3$

sub eq$^n$ (2).

$V_1 = I_1 \times \frac{5}{6}\times10^3$

$\frac{V_1}{I_1} = \frac{5}{6} k\Omega \Rightarrow \boxed{Z_{11} = \frac{5}{6} k\Omega}$

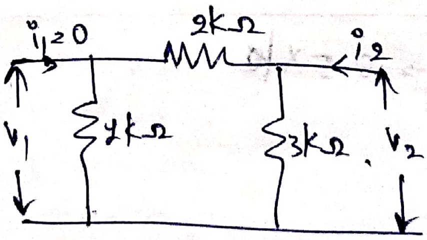
$I_{3k\Omega} = I_1 \times \frac{1\times10^3}{1\times10^3 + 2\times10^3 + 3\times10^3} = \frac{1\times10^3}{6\times10^3} = \frac{I_1}{6}$.

sub eq$^n$ (1).

$V_2 = \frac{I_1}{6} \times 3\times10^3 \Rightarrow \frac{V_2}{I_1} = \frac{10^3}{2} = 0.5\times10^3\,\Omega$.

$\boxed{Z_{21} = 0.5\times10^3\,\Omega}$ (or) $0.5 k\Omega$

Case (ii) let $i_1 = 0$.




$V_2 = I_2 \cdot R_{eq}$ — (1).

$V_1 = I_{1k\Omega} \times 1k$ — (2).

$R_{eq} = (1\times10^3 + 2\times10^3) \| 3\times10^3 = \frac{3\times10^3 \times 3\times10^3}{3\times10^3 + 3\times10^3} = \frac{9\times10^3}{6\times10^3} = \frac{3}{2}\times10^3$

$V_2 = I_2 \times \frac{3}{2} \times 10^3$

$\frac{V_2}{I_2} = 1.5\times10^3 \Rightarrow \boxed{Z_{22} = 1.5 k\Omega}$

$V_2 = I_{3k\Omega} \times 3k$

$$I_{1k\Omega} = I_2 \times \frac{3\times10^3}{1\times10^3+2\times10^3+3\times10^3} = I_2 \times \frac{3\times10^3}{6\times10^3}$$

$$I_{1k\Omega} = \frac{I_2}{2}$$

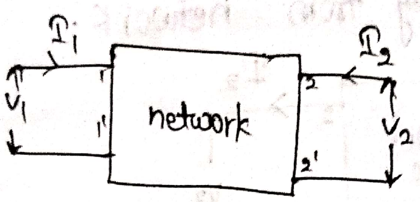
sub in eq - ①

$$V_1 = \frac{I_2}{2} \times 1k \Rightarrow V_1 = \frac{I_2}{2} \times 1\times10^3$$

$$\frac{V_1}{I_2} = \frac{10^3}{2} = 0.5\times10^3$$

$$\boxed{Z_{12} = 0.5\times10^3 \text{ (or) } 0.5k\Omega}$$

## h-parameters.




h-parameters are obtained by representing voltage at input port and current at output port, in the form of voltage at output port and current at input port.

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad -①$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad -②.$$

Case(i) :- let $I_1 = 0$.

from eq$^n$-①. $V_1 = h_{12} V_2 \Rightarrow \boxed{h_{12} = \frac{V_1}{V_2}}$

from eq$^n$-②: $I_2 = h_{22} V_2 \Rightarrow \boxed{h_{22} = \frac{I_2}{V_2}}$ ℧

Case (ii):- let $V_2 = 0$.

from eq$^n$① :- $V_1 = h_{11} I_1 \Rightarrow \boxed{h_{11} = \frac{V_1}{I_1}}$ Ω

from eq$^n$② :- $I_2 = h_{21} I_1 \Rightarrow \boxed{h_{21} = \frac{I_2}{I_1}}$

⇒ $h_{12}$ is called as open ckted reverse voltage gain.

$h_{22}$ is called as open ckted admittance.

$h_{11}$ is called as short ckted Impedance.

$h_{21}$ is called as short cked forward current gain.

h-parameters are also known as hybrid parameter.

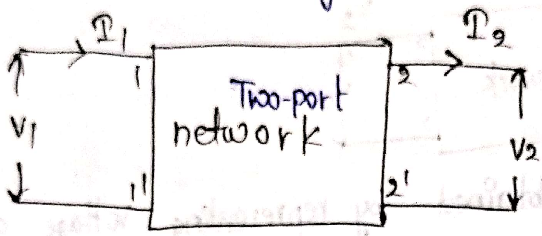
With respect to h-parameter, $V_1$ and $I_2$ are dependent parameters, whereas $V_2$, $I_1$ are Independent parameters.

## ABCD parameters

ABCD parameters are also called as Transmission parameters.

⇒ ABCD parameters can be obtained by representing voltage and current of input port interms of voltage and current of output port.

* ⇒ In the analysis of ABCD parameters direction of $I_2$ is assumed to be away from network.




$$V_1 = AV_2 + B(-I_2) \quad —①$$

$$I_1 = CV_2 + D(-I_2) \quad —②$$

Case(i) :- $-I_2 = 0$.

from eq^n ① :- $V_1 = AV_2 \Rightarrow \boxed{A = \frac{V_1}{V_2}}$

eq^n ② :- $I_1 = CV_2 \Rightarrow \boxed{C = \frac{I_1}{V_2}}$ ℧

Case (ii):- $V_2 = 0$.

from eq^n ① :- $V_1 = B(-I_2) \Rightarrow \boxed{B = \frac{-V_1}{I_2}}$ Ω

eq^n ②:- $I_1 = D(-I_2) \Rightarrow \boxed{D = -\frac{I_1}{I_2}}$

A is called as open ckted Reverse voltage gain.

C is called as open ckted Reverse admittance.

B is called as short ckted Reverse Impedance.

D is called as short circuited Reverse current gain.

Z-parameters.

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

y-parameters.

$$I_1 = y_{11} V_1 + y_{12} V_2$$
$$I_2 = y_{21} V_1 + y_{22} V_2$$

h-parameters.

$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$

ABCD parameters.

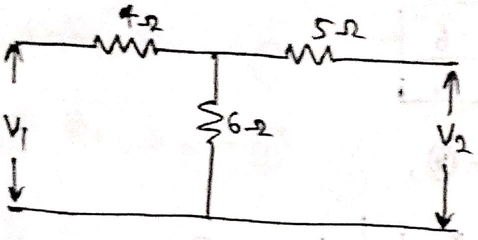
$$V_1 = AV_2 + B(-I_2)$$
$$I_1 = CV_2 + D(-I_2)$$

g-parameters.

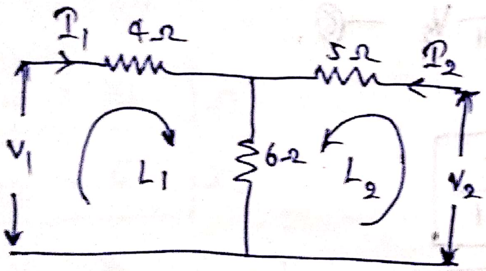
$$I_1 = g_{11} V_1 + g_{12} I_2$$
$$V_2 = g_{21} V_1 + g_{22} I_2$$

Problems.

1) find h-parameters for the circuit shown below.




Sol:-




Apply KVL to 'L1'

$$V_1 = 4i_1 + 6(i_1 + i_2)$$
$$V_1 = 10 i_1 + 6 i_2 \quad \text{—(1)}.$$

Apply KVL at 'L2'

$$V_2 = 5i_2 + 6(i_2 + i_1)$$
$$V_2 = 11 i_2 + 6 i_1$$
$$11 i_2 = V_2 - 6 i_1$$
$$i_2 = \frac{V_2}{11} - \frac{6}{11} i_1 \quad \text{—(2)}.$$

WKT h-parameters are from.

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{—(3)}$$
$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \text{—(4)}$$

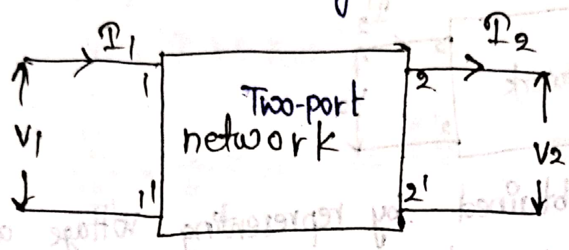
h-parameters are also known as hybrid parameter.
With respect to h-parameter, $V_1$ and $I_2$ are dependent parameters, whereas $V_2$, $I_1$ are Independent parameters.

## ABCD parameters

ABCD parameters are also called as Transmission parameters.

⇒ ABCD parameters can be obtained by representing voltage and current of input port interms of voltage and current at output port.

⇒ In the analysis of ABCD parameters direction of $I_2$ is assumed to be away from network.




$$V_1 = AV_2 + B(-I_2) \quad \text{—①}$$

$$I_1 = CV_2 + D(-I_2) \quad \text{—②}$$

case(i) :- $-I_2 = 0$.

from eqⁿ ① :- $V_1 = AV_2$ ⇒ $\boxed{A = \frac{V_1}{V_2}}$

eqⁿ ② :- $I_1 = CV_2$ ⇒ $\boxed{C = \frac{I_1}{V_2}}$ ℧

Case (ii) :- $V_2 = 0$.

from eqⁿ ① :- $V_1 = B(-I_2)$ ⇒ $\boxed{B = \frac{-V_1}{I_2}}$ Ω

eqⁿ ② :- $I_1 = D(-I_2)$ ⇒ $\boxed{D = \frac{-I_1}{I_2}}$

A is called as open ckted Reverse voltage gain.
C is called as open ckted Reverse admittance.
B is called as short ckted Reverse Impedance.
D is called as short circuited Reverse current gain.

Z-parameters.

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

Y-parameters.

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$
$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

h-parameters.

$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$

ABCD parameters.

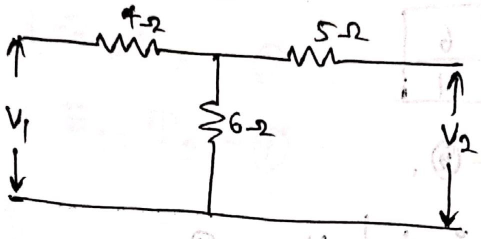
$$V_1 = AV_2 + B(-I_2)$$
$$I_1 = CV_2 + D(-I_2)$$

g-parameters.

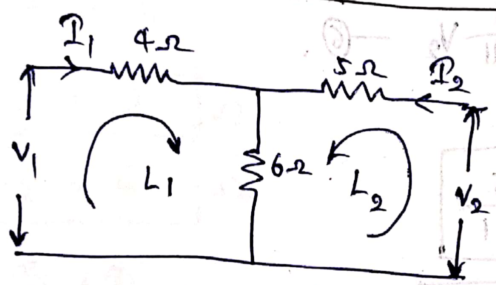
$$I_1 = g_{11} V_1 + g_{12} I_2$$
$$V_2 = g_{21} V_1 + g_{22} I_2$$

Problems.

1) find h-parameters for the circuit shown below.




Sol:-




Apply KVL to 'L1'

$$V_1 = 4i_1 + 6(i_1 + i_2)$$
$$V_1 = 10i_1 + 6i_2 \quad \text{—(1)}$$

Apply KVL at 'L2'

$$V_2 = 5i_2 + 6(i_2 + i_1)$$
$$V_2 = 11i_2 + 6i_1$$
$$11i_2 = V_2 - 6i_1$$
$$i_2 = \frac{V_2}{11} - \frac{6}{11} i_1 \quad \text{—(2)}.$$

WKT h-parameters are from.

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{—(3)}$$
$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \text{—(4)}$$

Sub eqⁿ ⑤ in ①.

$$V_1 = 10 i_1 + 6\left[\frac{V_2}{11} - \frac{6}{11} i_1\right]$$

$$V_1 = 10 i_1 + \frac{6}{11} V_2 - \frac{36}{11} i_1$$

$$V_1 = i_1\left[10 - \frac{36}{11}\right] + \frac{6}{11} V_2$$

$$V_1 = i_1 \frac{74}{11} + \frac{6}{11} V_2 \quad \text{— ⑤.}$$

Compare eqⁿ ③ & ⑤.

$$\boxed{h_{11} = \frac{74}{11}\,\Omega}$$

$$\boxed{h_{12} = \frac{6}{11}}$$

Rearrange eqⁿ — ②.

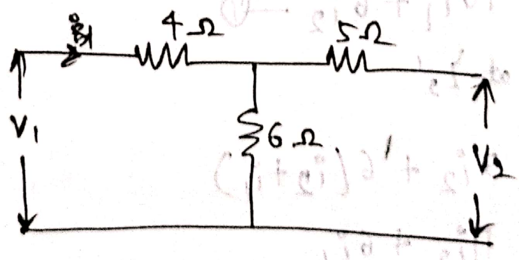
$$i_2 = \frac{-6}{11} i_1 + \frac{1}{11} V_2 \quad \text{— ⑥.}$$

Compare ⑥ & ④.

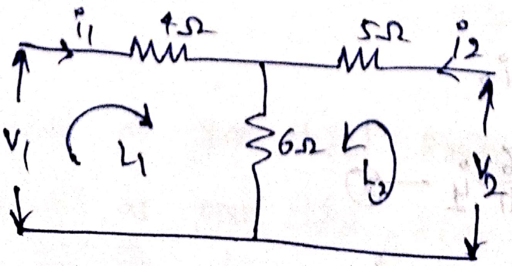
$$\boxed{h_{21} = \frac{-6}{11}}$$

$$\boxed{h_{22} = \frac{1}{11}\,\mho}$$

② Find z-parameters for the circuit shown below.




Sol:



let $i_2 = 0$.

Apply KVL at $L_1$

$$V_1 = 4i_1 + 6(i_1 + i_2)$$

$$V_1 = 10i_1 + 6i_2 \quad \text{—(1)}.$$

Apply KVL at $L_2$.

$$V_2 = 5i_1 + 6(i_2 + i_1)$$

$$V_2 = 11i_2 + 6i_1 \quad \text{—(2)}.$$

WKT $\quad = 6i_1 + 11i_2 \quad$ —(2).

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{—(3)}$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{—(4)}$$

Compare (1) & (3).

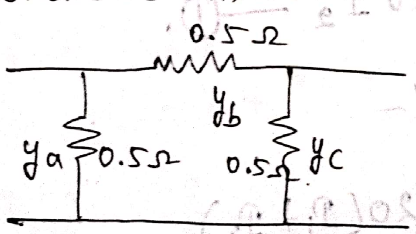
| | |
|---|---|
| $Z_{11} = 10$ | $\Omega$ |
| $Z_{12} = 6$ | $\Omega$ |

Compare (2) & (4)

| | |
|---|---|
| $Z_{21} = 6$ | $\Omega$ |
| $Z_{22} = 11$ | $\Omega$ |

27/11/2023

(3). For the two-port network shown in figure. find short circuit admittance parameters matrix.



Sol: Given $Y_a = 0.5\Omega$

$Y_b = 0.5\Omega$

$Y_c = 0.5\Omega$.

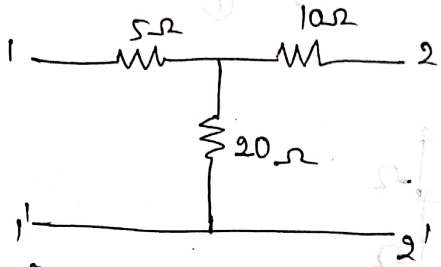
$Y_{11} = Y_a + Y_b = 0.5 + 0.5 = 1\ \mho$

$Y_{12} = Y_{21} = -Y_b = -0.5\ \mho$

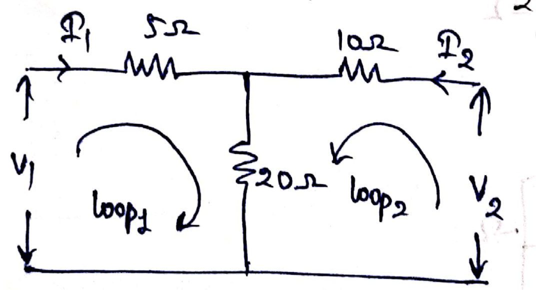
$Y_{22} = Y_b + Y_c = 0.5 + 0.5 = 1\ \mho$

$$Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} 1 & -0.5 \\ -0.5 & 1 \end{bmatrix}$$

② Find Z & Y parameters for the circuit shown below.




Sol:




Apply KVL at loop 1 :-

$$V_1 = 5I_1 + 20(I_1 + I_2).$$

$$= 25I_1 + 20I_2 \quad \text{—(1)}.$$

Apply KVL at loop 2 :-

$$V_2 = 10I_2 + 20(I_1 + I_2).$$

$$= 30I_2 + 20I_1$$

$$= 20I_1 + 30I_2 \quad \text{—(2)}$$

WKT for Z-parameters

$V_1 = Z_{11} I_1 + Z_{12} I_2$ — ③

$V_2 = Z_{21} I_1 + Z_{22} I_2$ — ④.

compare eqn ② & ③. compare eqn ② & ④.

| | |
|---|---|
| $Z_{11} = 25\,\Omega$ | $Z_{21} = 20\,\Omega$ |
| $Z_{12} = 20\,\Omega$ | $Z_{22} = 30\,\Omega$. |

Y-parameters.

from eqn — ①.

$$V_1 = 25 I_1 + 20 I_2$$

$$I_1 = \frac{1}{25} V_1 - \frac{20}{25} I_2$$

$$V_2 = 20 I_1 + 30 I_2$$

$$V_2 = 20\left[\frac{V_1}{25} - \frac{20}{25} I_2\right] + 30 I_2$$

$$V_2 = \frac{20V_1}{25} - \frac{400 I_2}{25} + 30 I_2$$

$$V_2 = \frac{20V_1}{25} + I_2\left[30 - \frac{400}{25}\right]$$

$$V_2 = \frac{20V_1}{25} + I_2 14$$

$$14 I_2 = \frac{-20}{25} V_1 + V_2$$

$$I_2 = \frac{-20}{25 \times 14} V_1 + \frac{1}{14} V_2$$

$$I_2 = \frac{-2}{35} V_1 + \frac{1}{14} V_2 \quad \text{— ⑤.}$$

$$I_1 = \frac{1}{25} V_1 - \frac{20}{25}\left[\frac{-2}{35} V_1 + \frac{1}{14} V_2\right]$$

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$$= \frac{1}{25} V_1 + \frac{20 \times 2}{25 \times 35} V_1 - \frac{20}{25 \times 14} V_2$$

$$I_1 = \frac{3}{35} V_1 - \frac{2}{35} V_2 \quad -(6)$$

$$I_2 = \frac{-2}{35} + \frac{1}{14} V_2 . \quad -(7)$$

Compare above eqn with

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \quad -(8)$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 . \quad -(9).$$

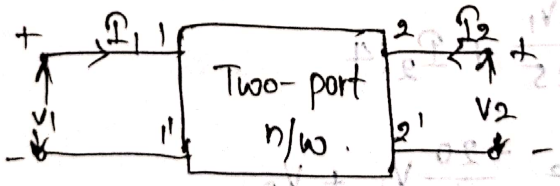
Compare (8) & (6).

$$\boxed{\begin{aligned} Y_{11} &= \frac{3}{35} \\ Y_{12} = Y_{21} &= -\frac{2}{35} \\ Y_{22} &= \frac{1}{14} \end{aligned}}$$

15/12/2023.

## g- parameters

g - parameters are also called as inverse of h-parameters and also conduction parameters.




current at input port (or) port-1 and voltage at output port (or) port-2 can be represented in the form of voltage at port-1 & current at port-2 with the help of g - parameters .

$$\boxed{\begin{aligned} I_1 &= g_{11}V_1 + g_{12}I_2 \quad -(1) \\ V_2 &= g_{21}V_1 + g_{22}I_2 \quad -(2) \end{aligned}}$$

from the above two eq^n^s it can be observed that $I_1$ and $V_2$ are dependent parameters, whereas $V_1$ and $I_2$ are Independent parameters with respect to g-parameters.

case (i):- let $V_1 = 0$ (short circuited).

from -(1): $I_1 = g_{12} I_2$

$$\boxed{g_{12} = \frac{I_1}{I_2}}$$

$g_{12}$ = short circuited reverse current gain.

from -(2): $V_2 = g_{22} I_2$

$$\boxed{g_{22} = \frac{V_2}{I_2}} \Omega$$

$g_{22} \rightarrow$ short circuited output impedance.

case (ii):- let $I_2 = 0$ (open circuited).

from -(1): $I_1 = g_{11} V_1$

$$\boxed{g_{11} = \frac{I_1}{V_1}} \mho$$

$g_{11} \rightarrow$ open circuited input admittance.

from -(2): $V_2 = g_{21} V_1$

$$\boxed{g_{21} = \frac{V_2}{I_1}}$$

$g_{21} \rightarrow$ open circuited forward voltage gain.

# Reciprocal Network

Def$^n$ :- A network is said to be reciprocal network, if the ratio of ~~exict~~ excitation to the response is same even if ports are interchanged.

Symmetrical network :- A network is said to be symmetrical network, if open circuited driving point impedance (or) short circuited driving point admittance is same at both the ports.

⇒ Conditions for reciprocal network and symmetrical network for Y-parameters.

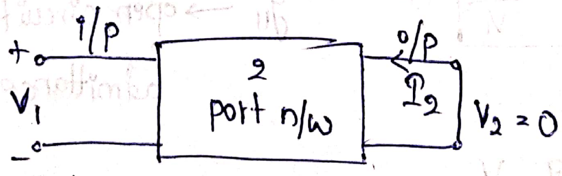
we know that a two-port network can be analyzed with the help of Y parameters by following.

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \quad \text{—}(1)$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 \quad \text{—}(2)$$

Reciprocal network:- A network is said to be reciprocal network, if the ratio of excitation to the response is same even if ports are interchanged.

Case (i): let the i/p be at port-①.




port -①  port ②.

If we take voltage as input, then take current as output by output voltage short ckted.

As per definition.

$$\left.\frac{V_1}{I_2}\right|_{V_2=0} \quad \text{—}(3)$$

At $V_2 = 0$

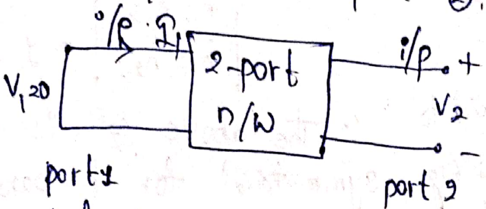
from-① $I_1 = Y_{11}V_1 \Rightarrow \boxed{V_1 = \frac{I_1}{Y_{11}}} \quad \text{—}(4)$.

from – ② :-

$$I_2 = Y_{21} V_1$$

$$\frac{V_1}{I_2} = \frac{1}{Y_{21}} \longrightarrow ⓐ.$$

case(2) :- let i/p be at port – ②.




As per definition

$$\left.\frac{V_2}{I_1}\right|_{V_1=0}$$

At $V_1 = 0$

from –① $I_1 = Y_{12} V_2$

$$\boxed{\frac{V_2}{I_1} = \frac{1}{Y_{12}}} \longrightarrow ⓑ.$$

As per Reciprocal n/w definition.

ⓐ = ⓑ

$$\frac{1}{Y_{21}} = \frac{1}{Y_{12}}$$

$$\boxed{Y_{12} = Y_{21}}$$

~~cond~~ Reciprocal condition for Y-parameters.

27/2/2023

Symmetrical network :- A network is said to be symmetrical if open circuited driving point impedance or short ckted driving point admittance is same at both the ports.

let us consider admittance to derive the condition :-

$$\boxed{Y_{11} = Y_{22}}$$

The above eqn is the symmetrical network condition for Y- parameters.

Conditions for Reciprocal network & symmetrical n/w for h-parameters.

WKT a 2 port n/w can be analysied with the help of h-parameters by following.

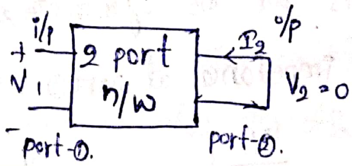
$$V_1 = h_{11} I_1 + h_{12} V_2 \quad —①$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad —②$$

Reciprocal n/w :- A n/w is said to be reciprocal n/w if the ratio of excitation to response same even after changing the ports.

Case-1 :- Let i/p be voltage ($V_1$) & o/p be current ($I_2$).

let i/p be at port ①.




$$\frac{i/p}{o/p} = \left.\frac{V_1}{I_2}\right|_{V_2=0} \quad —③.$$

if $V_2 = 0$

then $V_1 = h_{11} I_1$ —④

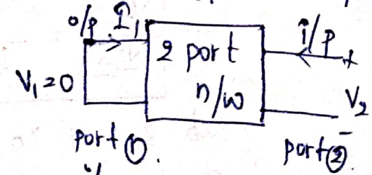
$I_2 = h_{21} I_1$ —⑤

Sub eqn ④ & ⑤ in eqn ③.

$$\frac{V_1}{I_2} = \frac{h_{11} \cancel{I_1}}{h_{21} \cancel{I_1}}$$

$$\boxed{\frac{V_1}{I_2} = \frac{h_{11}}{h_{21}}} — a$$

Case-2 :- let i/p be at port②




$$\frac{i/p}{o/p} = \left.\frac{V_2}{I_1}\right|_{V_1=0}$$

$$0 = h_{11} I_1 + h_{12} V_2 \quad —⑥$$

$$h_{12} V_2 = -h_{11} I_1$$

$$\boxed{\frac{V_2}{I_1} = \frac{-h_{11}}{h_{12}}} — ⓑ$$

As per Reciprocal n/w definition

eqn ⓐ = ⓑ

$$\frac{h_{11}}{h_{21}} = \frac{-h_{11}}{h_{12}}$$

$$\boxed{h_{12} = -h_{21}}$$

The above eqn is the reciprocal condition for h-parameters.

Symmetrical n/w :- A n/w is said to be symmetrical if open ckted driving point impedance or short ckted driving point admittance is same at both ports.

Let us consider impedance to derive condition.

As per the symmetrical n/w defn.

impedance at port① = impedance at port②.

$$\left.\frac{V_1}{I_1}\right|_{I_2=0} = \left.\frac{V_2}{I_2}\right|_{I_1=0} \quad — ⑦$$

at $I_2 = 0$

from eqn① $V_1 = h_{11} I_1 + h_{12} V_2$ — ⑧

from eqn② $0 = h_{21} I_1 + h_{22} V_2$

$h_{21} I_1 = -h_{22} V_2$

$$\boxed{I_1 = \frac{-h_{22}V_2}{h_{21}}} \quad — ⑨$$

at $I_1 = 0$

from eqn—① $V_1 = h_{12} V_2$ — ⑩

from eqn—② $I_2 = h_{22} V_2$

$$\boxed{\frac{V_2}{I_2} = \frac{1}{h_{22}}} \quad — ⑪$$

sub 8, 9, 11 in eqn—⑦.

$$\frac{h_{11}I_1 + h_{12}V_2}{\frac{-h_{22}V_2}{h_{21}}} = \frac{1}{h_{22}}$$

$$\frac{h_{21}[h_{11}I_1 + h_{12}V_2]}{-h_{22}V_2} = \frac{1}{h_{22}}$$

$$-h_{21}h_{11}I_1 + h_{21}h_{12}V_2 = V_2$$

$$-\left[h_{21}h_{11}\left(\frac{I_1}{V_2}\right) + h_{21}h_{12}(1)\right] = 1$$

$$-\left[h_{21}h_{11}\left(\frac{-h_{22}}{h_{21}}\right) + h_{21}h_{12}\right] = 1 \quad \text{(from eqn ⑨)}$$

$$\boxed{h_{11}h_{22} - h_{21}h_{12} = 1}$$

$$\begin{vmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{vmatrix} = 1$$

∴ The above eqn is condition of symmetrical for h-parameters.

Condition for Reciprocal n/w & Symmetrical n/w for ABCD parameters

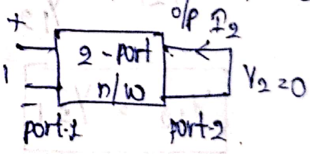
WKT a 2-port n/w can be analysied with the help of ABCD parameters.

$$V_1 = AV_2 + B(-I_2) \quad — ①$$

$$I_1 = CV_2 + D(-I_2) \quad — ②$$

Reciprocal n/w :- A n/w is said to be reciprocal n/w if the ratio of excitation to response is same in the ports are interchanged.

Case-1 :- let i/p be at port①.




$$\left.\frac{V_1}{I_2}\right|_{V_2=0} \quad — ③$$

at $V_2 = 0$

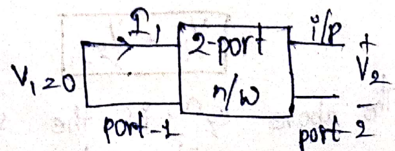
$V_1 = B(-I_2)$ — ④

$I_1 = -DI_2$ — ⑤

from eqn④

$$\boxed{\frac{V_1}{I_2} = -B} \quad — ⑥$$

Case-2 :- let i/p be at port②.



$$\frac{V_2}{I_1}\Big|_{V_1=0} \quad \text{—⑧}$$

at $V_1 = 0$

$0 = AV_2 - BI_2$ —⑥ $\quad V_2 = \frac{BI_2}{A}$

$I_1 = CV_2 - DI_2$ —⑦

Sub eq$^n$ ⑥ & ⑦ in eq$^n$ ⑧

$$\frac{V_2}{I_1} = \frac{\frac{BI_2}{A}}{CV_2 - DI_2}$$

$$= \frac{BI_2}{ACV_2 - DAI_2}$$

$$= \frac{B}{AC\left(\frac{V_2}{I_2}\right) - DA}$$

$$= \frac{B}{AC\left(\frac{B}{A}\right) - DA}$$

$$\boxed{\frac{V_2}{I_1} = \frac{B}{CB - DA}} \text{—⑤}$$

from def$^n$ of Reciprocal n/w ⓐ = ⓑ.

$$\frac{-B}{1} = \frac{B}{CB - DA}$$

$$-(CB - DA) = 1$$

$$-CB + DA = 1$$

$$DA - CB = 1 \quad \therefore \boxed{AD - BC = 1}$$

∴ The above eq$^n$ is the condition for reciprocal n/w of ABCD parameters.

Symmetrical n/w :- A n/w is said to be symmetrical if open ckted driving point impedance or short ckted driving point admittance is same at both ports.

Let us consider impedance to derive condition.

As per the symmetrical n/w def$^n$.

impedance at port ① = impedance at port ②.

$$\frac{V_1}{I_1}\Big|_{I_2=0} = \frac{V_2}{I_2}\Big|_{I_1=0} \text{—⑨.}$$

at $I_2 = 0$

$V_1 = AV_2$ —⑩

$I_1 = CV_2$ —⑪

sub eq$^n$ ⑩, ⑪ in eq$^n$ ⑨

$$\frac{V_1}{I_1} = \frac{AV_2}{CV_2}$$

$$\boxed{\frac{V_1}{I_1} = \frac{A}{C}} \text{—ⓐ}$$

at $I_1 = 0$

$V_1 = AV_2 - BI_2$

$0 = CV_2 - DI_2$

$CV_2 = DI_2$

$$\boxed{\frac{V_2}{I_2} = \frac{D}{C}} \text{—ⓑ}$$

Sub eq$^n$ a, b in eq$^n$ ⑨.

$$\frac{A}{C} = \frac{D}{C}$$

$$\therefore \boxed{A = D}$$

∴ The above eq$^n$ is the condition for symmetrical n/w for ABCD parameters.

03/1/2024

Conditions for Reciprocal n/w conditions and symmetrical n/w conditions for g-parameters.

W.K.T for g parameters

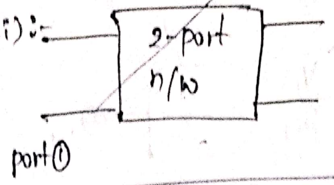
$$I_1 = g_{11}V_1 + g_{12}I_2 \text{—①}$$

$$I_2 = g_{21}V_1 + g_{22}I_2 \text{—②.}$$

Reciprocal n/w :- A n/w is said to be reciprocal n/w if the ratio of excitation to response is same in the ports are interchanged.

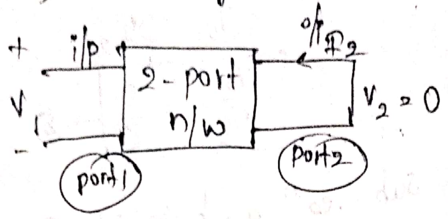
~~Case(ii). let i/p be voltage & o/p be current.~~

Case(i):-




let i/p be voltage & o/p be current.

case (i). let voltage (or) i/p be at port 1.




$$\frac{\text{excitation}}{\text{response}} = \left.\frac{V_1}{I_2}\right|_{V_2=0} \quad \text{—③}$$

At $V_2 = 0$

from ①.

$$I_1 = g_{11}V_1 + g_{12}I_2 \quad \text{—④.}$$

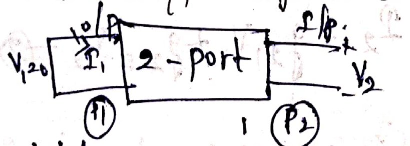
from ②.

$$0 = g_{21}V_1 + g_{22}I_2$$

$$g_{21}V_1 = -g_{22}I_2$$

$$\boxed{\frac{V_1}{I_2} = \frac{-g_{22}}{g_{21}}} \quad \text{—ⓐ.}$$

Case (ii) :- let i/p voltage be at port②




$$\frac{\text{excitation}}{\text{response}} = \left.\frac{V_2}{I_1}\right|_{V_1=0} \quad \text{—⑤}$$

At $V_1 = 0$

from ①

$$I_1 = g_{12}I_2 \quad \text{—⑥.}$$

from ②.

$$V_2 = g_{22}I_2 \quad \text{—⑦}$$

sub ⑥, ⑦ in ⑤.

$$\frac{V_2}{I_1} = \frac{g_{22}\cancel{I_2}}{g_{12}\cancel{I_2}}$$

$$\boxed{\frac{V_2}{I_1} = \frac{g_{22}}{g_{12}}} \quad \text{—ⓑ.}$$

As per definition

ⓐ = ⓑ.

$$\boxed{\frac{-\cancel{g_{22}}}{g_{21}} = \frac{\cancel{g_{22}}}{g_{12}}} \Rightarrow \boxed{g_{12} = -g_{21}}$$

∵ The above eqn is the reciprocal condition for g-parameters.

Symmetrical n/w :- A n/w is said to be symmetrical if open ckted driving point impedance or short ckted driving point admittance is same at both ports.

→ let us consider impedance for the analysis.

As per the defn of symmetrical n/w.

impedance at port① = impedance at port②

$$\left.\frac{V_1}{I_1}\right|_{I_2=0} = \left.\frac{V_2}{I_2}\right|_{I_1=0} \quad \text{—⑧.}$$

let $I_2 = 0$

from eqn ①.

$$I_1 = g_{11}V_1$$

$$\boxed{\frac{V_1}{I_1} = \frac{1}{g_{11}}} \quad \text{—⑨.}$$

let $I_1 = 0$

from ① $0 = g_{11}V_1 + g_{12}I_2$

$g_{12} I_2 = -g_{11} V_1$

$$I_2 = \frac{-g_{11}V_1}{g_{12}} \quad \text{— (10).}$$

from – (2)

$$V_2 = g_{21}V_1 + g_{22} I_2 \quad \text{— (11).}$$

sub (9), (10), (11) in (8).

$$\frac{1}{g_{11}} = \frac{g_{21}V_1 + g_{22}I_2}{\frac{-g_{11}V_1}{g_{12}}}$$

$$\frac{1}{g_{11}} = \frac{g_{12}g_{21}V_1 + g_{12}g_{22}I_2}{-g_{11}V_1}$$

$$\frac{1}{g_{11}} = \frac{-1}{g_{11}}\left[\frac{g_{12}g_{21}\cancel{V_1}}{\cancel{V_1}} + \frac{g_{12}g_{21}I_2}{V_1}\right]$$

$$\frac{1}{g_{11}} = \frac{-1}{g_{11}}\left[g_{12}g_{21} + \cancel{g_{12}}g_{22}\left(\frac{-g_{11}}{\cancel{g_{12}}}\right)\right] \quad \text{(from (10))}$$

$$1 = -g_{12}g_{21} + g_{11}g_{22}$$

$$\boxed{g_{11}g_{22} - g_{12}g_{21} = 1 \qquad \begin{vmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{vmatrix} = 1}$$

∴ The above eqn is the symmetrical condition for g-parameters.

Conditions for reciprocal n/w and symmetrical n/w for Z-parameters.

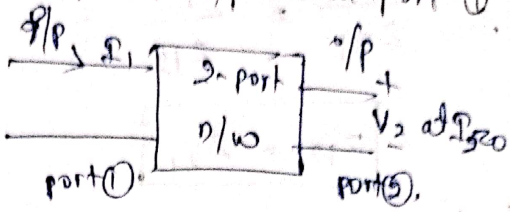
WKT. $V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2$.

Reciprocal n/w :- A n/w is said to be reciprocal n/w if the ratio of excitation to response is same if the ports are interchanged.

let current be the i/p and Voltage be the o/p.

Case (i) :- let i/p be at port-(1)




$$\frac{\text{excitation}}{\text{Response}} = \frac{I_1}{V_2}\bigg|_{I_2=0}$$

at $I_2 = 0$.

from – (1)

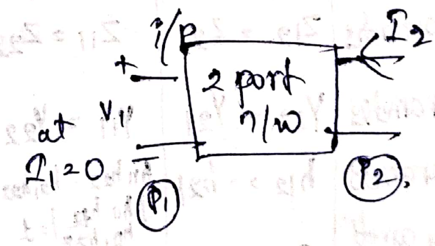
$V_1 = Z_{11} I_1 \Rightarrow I_1 = \frac{V_1}{Z_{11}}$ — (3)

from – (2).

$V_2 = Z_{21} I_1$

$$\boxed{\frac{I_1}{V_2} = \frac{1}{Z_{21}}} \quad \text{— (a).}$$

Case (2) :- let i/p be at port-2




$$\frac{\text{excitation}}{\text{Response}} = \frac{I_2}{V_1}\bigg|_{I_1=0}$$

at $I_2 = 0$

from – (1).

$V_1 = Z_{12} I_2$

$$\boxed{\frac{1}{Z_{12}} = \frac{I_2}{V_1}} \quad \text{— (b)}$$

As per def$^n$ of reciprocal n/w.

(a) = (b).

$$\frac{1}{Z_{21}} = \frac{1}{Z_{12}}$$

$$\boxed{Z_{12} = Z_{21}}$$

The above eq$^n$ is the reciprocal condition for Z-parameter.

Symmetrical n/w :- A n/w is said to be symmetrical if open ckt'd impedance (or) short ckt'd driving point admittance is same at both ports.

As per the def$^n$ $\boxed{Z_{11} = Z_{22}}$ //.

Conditions for

| Parameters. | reciprocal n/w | Symmetrical n/w |
|---|---|---|
| Z-parameters | $Z_{12} = Z_{21}$ | $Z_{11} = Z_{22}$ |
| Y-parameters | $Y_{12} = Y_{21}$ | $Y_{11} = Y_{22}$ |
| h-parameters | $h_{12} = -h_{21}$ | $h_{11}h_{22} - h_{21}h_{12} = 1$ <br> $\begin{vmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{vmatrix} = 1$ |
| g-parameters | $g_{12} = -g_{21}$ | " |
| ABCD | $AD - BC = 1$ | $A = D$. |

→ Conversion of Z-parameters.

Z-parameter to Y-parameters.

WKT

$[Z] = [Y]^{-1}$ where $[Y] = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$

$$[Z] = \frac{\text{Adj } [Y]}{|Y|}$$

= Transpose of [Y], with element replaced its co-factors / $|Y|$

$$[Z] = \frac{\begin{bmatrix} Y_{22} & -Y_{21} \\ -Y_{12} & Y_{11} \end{bmatrix}^T}{|Y|}$$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \frac{\begin{bmatrix} Y_{22} & -Y_{12} \\ -Y_{21} & Y_{11} \end{bmatrix}}{|Y|}$$

$$\boxed{Z_{11} = \frac{Y_{22}}{|Y|}, \quad Z_{12} = \frac{-Y_{12}}{|Y|} \\ Z_{21} = \frac{-Y_{21}}{|Y|}, \quad Z_{22} = \frac{Y_{11}}{|Y|}}$$

where

$|Y| = Y_{11}Y_{22} - Y_{12}Y_{21}$

→ Z-parameters interms of h-parameters.

WKT for Z-parameters.

$V_1 = Z_{11}I_1 + Z_{12}I_2$ —(1)

$V_2 = Z_{21}I_1 + Z_{22}I_2$ —(2)

WKT for h-parameters.

$V_1 = h_{11}I_1 + h_{12}V_2$ —(3)

$I_2 = h_{21}I_1 + h_{22}V_2$ —(4)

from (4)

$h_{22}V_2 = I_2 - h_{21}I_1$

$$V_2 = \frac{I_2}{h_{22}} - \frac{h_{21}}{h_{22}} I_1$$

$$V_2 = \left(\frac{-h_{21}}{h_{22}}\right) I_1 + \left(\frac{1}{h_{22}}\right) I_2 \quad -(5)$$

Sub (5) in (3)

$$V_1 = h_{11} I_1 + h_{12}\left[\frac{-h_{21} I_1}{h_{22}} + \frac{I_2}{h_{22}}\right]$$

$$= h_{11} I_1 - \frac{h_{12} h_{21} I_1}{h_{22}} + \frac{h_{12} I_2}{h_{22}}$$

$$V_1 = I_1\left[h_{11} - \frac{h_{12} h_{21}}{h_{22}}\right] + \left(\frac{h_{12}}{h_{22}}\right) I_2 \quad -(6)$$

Compare (1) & (6), (2) & (5)

$$\boxed{\begin{aligned} Z_{11} &= h_{11} - \frac{h_{12} h_{21}}{h_{22}} \\ Z_{12} &= \frac{h_{12}}{h_{22}} \\ Z_{21} &= \frac{-h_{21}}{h_{22}} ; \quad Z_{22} = \frac{1}{h_{22}} \end{aligned}}$$

→ Z-parameters interms of g-parameters

WKT for Z-parameters

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad -(1)$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad -(2).$$

WKT for g-parameters

$$I_1 = g_{11} V_1 + g_{12} I_2 \quad -(3)$$

$$V_2 = g_{21} V_1 + g_{22} I_2 \quad -(4)$$

from eqn -(3)

$$g_{11} V_1 = I_1 - g_{12} V_2$$

$$V_1 = \frac{I_1}{g_{11}} - \frac{g_{12} I_2}{g_{11}} \quad -(5).$$

Sub eqn (5) in (4)

$$V_2 = g_{21}\left[\frac{I_1}{g_{11}} - \frac{g_{12} I_2}{g_{11}}\right] + g_{22} I_2$$

$$V_2 = \frac{g_{21} I_1}{g_{11}} - \frac{g_{21} g_{12} I_2}{g_{11}} + g_{22} I_2$$

$$V_2 = \frac{g_{21} I_1}{g_{11}} + I_2\left[g_{22} - \frac{g_{21} g_{12}}{g_{11}}\right] \quad -(6)$$

compare (1) & (5), (2) & (6)

$$\boxed{\begin{aligned} Z_{11} &= \frac{1}{g_{11}}, \quad Z_{12} = -\frac{g_{12}}{g_{11}} \\ Z_{21} &= \frac{g_{21}}{g_{11}}, \quad Z_{22} = g_{22} - \frac{g_{21} g_{12}}{g_{11}} \end{aligned}}$$

Z-parameters interms of ABCD parameters.

WKT for Z-parameters.

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad -(1)$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad -(2)$$

WKT for ABCD parameters

$$V_1 = A V_2 - B I_2 \quad -(3)$$

$$I_1 = C V_2 - D I_2 \quad -(4)$$

from eqn (4)

$$C V_2 = I_1 + D I_2$$

$$V_2 = \frac{I_1}{C} + \frac{D I_2}{C} \quad -(5)$$

Sub (5) in (3)

$$V_1 = A\left[\frac{I_1}{C} + \frac{D I_2}{C}\right] - B I_2$$

$$V_1 = \frac{A I_1}{C} + \frac{A D I_2}{C} - B I_2$$

$$V_1 = \frac{A}{C} I_1 + I_2\left[\frac{AD}{C} - B\right] \quad -(6)$$

Compare (1) & (6), (2) & (5)

$$\boxed{\begin{aligned} Z_{11} &= \frac{A}{C}, \quad Z_{21} = \frac{1}{C} \\ Z_{12} &= \frac{AD}{C} - B, \quad Z_{22} = \frac{D}{C} \end{aligned}}$$

Conversion of Y- parameters.

⇒ Y-parameters interms of z-parameters.

WKT

$[Y] = [Z]^{-1}$ where $\therefore |Z| = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$

$$[Y] = \frac{Adj[Z]}{|Z|}$$

$$\frac{\text{Transpose of } [Z]\text{, with elements replaced with its co-factors}}{|Z|}$$

$$[Y] = \frac{\begin{bmatrix} Z_{22} & -Z_{21} \\ -Z_{12} & Z_{11} \end{bmatrix}^T}{|Z|}$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \frac{\begin{bmatrix} Z_{22} & -Z_{12} \\ -Z_{21} & Z_{11} \end{bmatrix}}{|Z|}$$

$$\boxed{\begin{aligned} Y_{11} &= \frac{Z_{22}}{|Z|}, & Y_{12} &= \frac{-Z_{12}}{|Z|} \\ Y_{21} &= \frac{-Z_{21}}{|Z|}, & Y_{22} &= \frac{Z_{11}}{|Z|} \end{aligned}}$$

where.

$$|Z| = Z_{11}Z_{22} - Z_{12}Z_{21}$$

Y- parameters interms of h- parameters.

WKT Y- parameters.

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \quad -(1)$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 \quad -(2)$$

WKT h- parameters.

$$V_1 = h_{11}I_1 + h_{12}V_2 \quad -(3)$$

$$I_2 = h_{21}I_1 + h_{22}V_2 \quad -(4)$$

from eq^n (3)

$$h_{11}I_1 = V_1 - h_{12}V_2$$

$$I_1 = \frac{V_1}{h_{11}} - \frac{h_{12}}{h_{11}}V_2 \quad -(5)$$

Sub (5) in (4)

$$I_2 = h_{21}\left(\frac{V_1}{h_{11}} - \frac{h_{12}}{h_{11}}V_2\right) + h_{22}V_2$$

$$I_2 = \frac{h_{21}V_1}{h_{11}} - \frac{h_{21}h_{12}}{h_{11}}V_2 + h_{22}V_2$$

$$I_2 = \frac{h_{21}}{h_{11}}V_1 - V_2\left[\frac{h_{21}h_{12}}{h_{11}} + h_{22}\right]$$

Compare (1) & (5), (2) & (6).

$$\boxed{\begin{aligned} Y_{11} &= \frac{1}{h_{11}}, & Y_{12} &= \frac{-h_{12}}{h_{11}} \\ Y_{21} &= \frac{h_{21}}{h_{11}}, & Y_{22} &= -\frac{h_{21}h_{12}}{h_{11}} + h_{22} \end{aligned}}$$

Y- parameters interms of g- parameters.

WKT Y- parameters.

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \quad -(1)$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 \quad -(2)$$

WKT g- parameters.

$$I_1 = g_{11}V_1 + g_{12}I_2 \quad -(3)$$

$$V_2 = g_{21}V_1 + g_{22}I_2 \quad -(4)$$

from eqⁿ - (4)

$$g_{22} I_2 = V_2 - g_{21} V_1$$

$$I_2 = \frac{V_2}{g_{22}} - \frac{g_{21} V_1}{g_{22}}$$

$$I_2 = -\frac{g_{21}}{g_{22}} V_1 + \frac{V_2}{g_{22}} \quad -(5)$$

sub (5) in (3).

$$I_1 = g_{11} V_1 + g_{12}\left[-\frac{g_{21}}{g_{22}} V_1 + \frac{V_2}{g_{22}}\right]$$

$$I_1 = g_{11} V_1 + \frac{g_{12}\, g_{21}}{g_{22}} V_1 + \frac{V_2}{g_{22}}$$

$$I_1 = V_1\left[g_{11} - \frac{g_{12} g_{21}}{g_{22}}\right] + \frac{V_2}{g_{22}} \quad -(6)$$

Compare (1) & (6), (2) & (5).

$$\boxed{\begin{aligned} Y_{11} &= g_{11} - \frac{g_{12} g_{21}}{g_{22}} \\ Y_{12} &= \frac{1}{g_{22}} \\ Y_{21} &= -\frac{g_{21}}{g_{22}}, \quad Y_{22} = \frac{1}{g_{22}} \end{aligned}}$$

Y-parameters interms of ABCD parameters.

WKT Y-parameters.

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \quad -(1)$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 \quad -(2)$$

WKT ABCD-parameters.

$$V_1 = A V_2 - B I_2 \quad -(3)$$

$$I_1 = C V_2 - D I_2 \quad -(4)$$

from eqⁿ - (3)

$$B I_2 = A V_2 - V_1$$

$$I_2 = \frac{A}{B} V_2 - \frac{1}{B} V_1$$

$$I_2 = -\frac{1}{B} V_1 + \frac{A}{B} V_2 \quad -(5)$$

sub (5) in (4).

$$I_1 = C V_2 - D\left(-\frac{1}{B} V_1 + \frac{A}{B} V_2\right)$$

$$I_1 = C V_2 + \frac{D}{B} V_1 - \frac{DA}{B} V_2$$

$$I_1 = \frac{D}{B} V_1 + V_2\left(C - \frac{AD}{B}\right) \quad -(6)$$

Compare (1) & (6), (2) & (5).

$$\boxed{\begin{aligned} Y_{11} &= \frac{D}{B}, \quad Y_{12} = C - \frac{AD}{B} \\ Y_{21} &= -\frac{1}{B}, \quad Y_{22} = \frac{A}{B} \end{aligned}}$$

⇒ Conversions of h-parameters.

~~WKT~~ h parameters interms of Z-parameters.

WKT. h-parameters.

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad -(1)$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad -(2)$$

WKT. Z-parameters.

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad -(3)$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad -(4)$$

from eqⁿ - (4).

$$Z_{22} I_2 = V_2 - Z_{21} I_1$$

$$I_2 = \frac{V_2}{Z_{22}} - \frac{Z_{21}}{Z_{22}} I_1$$

$$I_2 = -\frac{Z_{21}}{Z_{22}} I_1 + \frac{1}{Z_{22}} V_2 \quad -(5)$$

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Sub ⑤ in ③.

$$V_1 = Z_{11}I_1 + Z_{12}\left[-\frac{Z_{21}}{Z_{22}}I_1 + \frac{I_2}{Z_{22}}\right]$$

$$V_1 = Z_{11}I_1 + \frac{Z_{12}Z_{21}}{Z_{22}}I_1 + \frac{Z_{12}}{Z_{22}}I_2$$

$$V_1 = \left[Z_{11} - \frac{Z_{12}Z_{21}}{Z_{22}}\right]I_1 + \frac{Z_{12}}{Z_{22}}I_2 \quad \text{—⑥}$$

Compare ① & ⑥, ② & ⑤

$$\boxed{h_{11} = Z_{11} - \frac{Z_{12}Z_{21}}{Z_{22}}, \quad h_{12} = \frac{Z_{12}}{Z_{22}} \\ h_{21} = -\frac{Z_{21}}{Z_{22}}, \quad h_{22} = \frac{1}{Z_{22}}}$$

h-parameters interms of Y-parameters.

WKT h-parameters

$$V_1 = h_{11}I_1 + h_{12}V_2 \quad \text{—①}$$

$$I_2 = h_{21}I_1 + h_{22}V_2 \quad \text{—②}$$

WKT Y-parameters.

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \quad \text{—③}$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 \quad \text{—④}$$

from eqn ④.

$$Y_{11}V_1 = I_1 - Y_{12}V_2$$

$$V_1 = \frac{I_1}{Y_{11}} - \frac{Y_{12}}{Y_{11}}V_2 \quad \text{—⑤}$$

Sub ⑤ in ④

$$I_2 = Y_{21}\left[\frac{I_1}{Y_{11}} - \frac{Y_{12}}{Y_{11}}V_2\right] + Y_{22}V_2$$

$$I_2 = \frac{Y_{21}I_1}{Y_{11}} - \frac{Y_{21}Y_{12}}{Y_{11}}V_2 + Y_{22}V_2$$

$$I_2 = \frac{Y_{21}}{Y_{11}}I_1 + V_2\left[Y_{22} - \frac{Y_{21}Y_{12}}{Y_{11}}\right] \quad \text{—⑥}$$

Compare ① & ⑤, ② & ⑥.

$$\boxed{h_{11} = \frac{1}{Y_{11}}, \quad h_{12} = -\frac{Y_{12}}{Y_{11}} \\ h_{21} = \frac{Y_{21}}{Y_{11}}, \quad h_{22} = Y_{22} - \frac{Y_{21}Y_{12}}{Y_{11}}}$$

h-parameters interms of g-parameters.

WKT h-parameters.

$$V_1 = h_{11}I_1 + h_{12}V_2 \quad \text{—①}$$

$$I_2 = h_{21}I_1 + h_{22}V_2 \quad \text{—②}$$

WKT g-parameters.

$$I_1 = g_{11}V_1 + g_{12}I_2 \quad \text{—③}$$

$$V_2 = g_{21}V_1 + g_{22}I_2 \quad \text{—④}$$

WKT

$$[h] = [g]^{-1}$$

$$= \frac{\text{adj}[g]}{|g|} \qquad g = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}$$

$$[h] = \frac{\text{Transponse of } g \text{ with the elements replaced by its co factors}}{|g|}$$

$$[h] = \frac{\begin{bmatrix} g_{22} & -g_{21} \\ -g_{12} & g_{11} \end{bmatrix}^T}{|g|}$$

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \frac{\begin{bmatrix} g_{22} & -g_{21} \\ -g_{12} & g_{22} \end{bmatrix}}{|g|}$$

$$\boxed{h_{11} = \frac{g_{22}}{|g|}, \quad h_{12} = \frac{-g_{21}}{|g|} \\ h_{21} = \frac{-g_{12}}{|g|}, \quad h_{22} = \frac{g_{22}}{|g|}}$$

Here $|g| = g_{11}g_{22} - g_{12}g_{21}$

h-parameters interms of ABCD parameters.

WKT h-parameters.

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad -① $$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad -② $$

WKT ABCD parameters.

$$V_1 = A V_2 - B I_2 \quad -③ $$

$$I_1 = C V_2 - D I_2 \quad -④ $$

from eq$^n$ ④.

$$D I_2 = C V_2 - I_1$$

$$I_2 = \frac{C}{D} V_2 - \frac{1}{D} I_1$$

$$I_2 = -\frac{1}{D} I_1 + \frac{C}{D} V_2 \quad -⑤ .$$

sub ⑤ in ③.

$$V_1 = A V_2 - B\left[-\frac{1}{D} I_1 + \frac{C}{D} V_2\right]$$

$$V_1 = A V_2 + \frac{B}{D} I_1 - \frac{BC}{D} V_2 .$$

$$V_1 = \frac{B}{D} I_1 + V_2\left[A - \frac{BC}{D}\right] \quad -⑥ .$$

compare ① & ⑥, ② & ⑤.

$$\boxed{h_{11} = \frac{B}{D}, \quad h_{12} = A - \frac{BC}{D} \qquad h_{21} = -\frac{1}{D}, \quad h_{22} = \frac{C}{D}}$$

**Conversion of g-parameters.**

g-parameters interms of Z-parameters.

WKT g-parameters.

$$I_1 = g_{11} V_1 + g_{12} I_2 \quad -① $$

$$V_2 = g_{21} V_1 + g_{22} I_2 \quad -② .$$

WKT Z-parameters.

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad -③ $$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad -④ $$

from eq$^n$ ③.

$$Z_{11} I_1 = V_1 - Z_{12} I_2$$

$$I_1 = \frac{V_1}{Z_{11}} - \frac{Z_{12}}{Z_{11}} I_2 \quad -⑤ $$

sub ⑤ in ④.

$$V_2 = Z_{21}\left[\frac{V_1}{Z_{11}} - \frac{Z_{12}}{Z_{11}} I_2\right] + Z_{22} I_2$$

$$V_2 = \frac{Z_{21} V_1}{Z_{11}} - \frac{Z_{21} Z_{12}}{Z_{11}} I_2 + Z_{22} I_2 .$$

$$V_2 = \frac{Z_{21}}{Z_{11}} V_1 + I_2\left[Z_{22} - \frac{Z_{21} Z_{12}}{Z_{11}}\right] \quad -⑥ $$

compare ① & ⑤, ② & ⑥.

$$\boxed{g_{11} = \frac{1}{Z_{11}}, \quad g_{12} = -\frac{Z_{12}}{Z_{11}} \qquad g_{21} = \frac{Z_{21}}{Z_{11}}, \quad g_{22} = \left[Z_{22} - \frac{Z_{21} Z_{12}}{Z_{11}}\right]}$$

g-parameters interms of Y-parameters.

WKT g-parameters.

$$I_1 = g_{11} V_1 + g_{12} I_2 \quad -① $$

$$V_2 = g_{21} V_1 + g_{22} I_2 \quad -② .$$

WKT Y-parameters.

$$I_1 = Z_{11} V_1 + Z_{12} V_2 \quad -③ $$

$$I_2 = Z_{21} V_1 + Z_{22} V_2 \quad -④ $$

from eq$^n$ ④.

$$Z_{22} V_2 = Z_{21} V_1 - I_2$$

$$V_2 = \frac{Z_{21}}{Z_{22}} V_1 - \frac{1}{Z_{22}} I_2 \quad -⑤ $$

sub (5) in (3).

$$I_1 = Z_{11}V_1 + Z_{12}\left[\frac{Z_{21}}{Z_{22}}V_1 - \frac{1}{Z_{22}}I_2\right]$$

$$I_1 = Z_{11}V_1 + \frac{Z_{12}Z_{21}}{Z_{22}}V_1 - \frac{Z_{12}}{Z_{22}}I_2$$

$$I_1 = \left[Z_{11} + \frac{Z_{12}Z_{21}}{Z_{22}}\right]V_1 - \frac{Z_{12}}{Z_{22}}I_2 \quad -(6).$$

Compare (1)&(6), (2)&(5).

$$\boxed{g_{11} = Z_{11} + \frac{Z_{12}Z_{21}}{Z_{22}}, \quad g_{12} = -\frac{Z_{12}}{Z_{22}}, \quad g_{21} = \frac{Z_{21}}{Z_{22}}, \quad g_{22} = -\frac{1}{Z_{22}}}$$

g-parameters interms of h-parameters.

WKT g-parameters.

$$I_1 = g_{11}V_1 + g_{12}I_2 \quad -(1)$$

$$V_2 = g_{21}V_1 + g_{22}I_2 \quad -(2)$$

WKT h-parameters.

$$V_1 = h_{11}I_1 + h_{12}V_2 \quad -(3)$$

$$I_2 = h_{21}I_1 + h_{22}V_2 \quad -(4).$$

from eq$^n$ (3).

$$h_{11}I_1 = V_1 - h_{12}V_2$$

$$I_1 = \frac{V_1}{h_{11}} - \frac{h_{12}}{h_{11}}V_2$$

ABCD interms of Z parameters.

$$A = \frac{Z_{11}}{Z_{21}}$$

$$B = -Z_{12} + \frac{Z_{11}Z_{22}}{Z_{21}} \text{ (or) } \frac{Z_{11}Z_{22}}{Z_{21}} - Z_{12}$$

$$C = \frac{1}{Z_{21}}, \quad D = \frac{-Z_{22}}{Z_{21}}$$

ABCD interms Y-p.

$$A = \frac{1}{Y_{21}} \qquad C = \frac{Y_{11}}{Y_{21}}$$

$$B = \frac{Y_{22}}{Y_{12}} \qquad D = \frac{Y_{11}Y_{22}}{Y_{12}} - Y_{12}$$

g-parameters interms of ABCD parameters.

WKT g-parameters.

$$I_1 = g_{11}V_1 + g_{12}I_2 \quad -(1)$$

$$V_2 = g_{21}V_1 + g_{22}I_2 \quad -(2)$$

WKT ABCD parameters

$$V_1 = AV_2 - BI_2 \quad -(3)$$

$$I_1 = CV_2 - DI_2 \quad -(4)$$

from eq$^n$ (3).

$$AV_2 = V_1 + BI_2$$

$$V_2 = \frac{V_1}{A} + \frac{B}{A}I_2 \quad -(5).$$

Sub (5) in (4).

$$I_1 = C\left[\frac{V_1}{A} + \frac{B}{A}I_2\right] - DI_2$$

$$I_1 = \frac{CV_1}{A} + \frac{CB}{A}I_2 - DI_2$$

$$I_1 = \frac{C}{A}V_1 + I_2\left[\frac{CB}{A} - D\right] \quad -(6)$$

compare (1)&(6), (2)&(5).

$$\boxed{g_{11} = \frac{C}{A}, \quad g_{12} = \frac{CB}{A} - D, \quad g_{21} = \frac{1}{A}, \quad g_{22} = \frac{B}{A}}$$

ABCD interms h-parameters.

$$A = \frac{h_{11}}{h_{21}} \qquad C = \frac{1}{h_{21}}$$

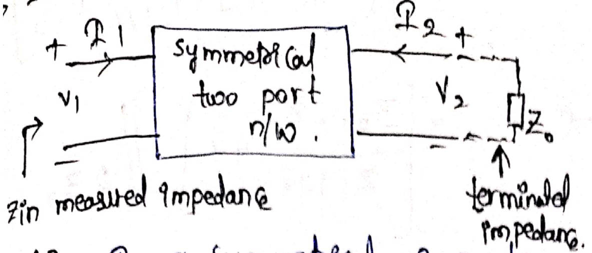
$$B = -h_{12} + \frac{h_{22}}{h_{21}} \qquad D = \frac{h_{22}}{h_{21}}$$

ABCD interms g parameters.

$$A = \frac{1}{g_{21}} \qquad C = \frac{g_{11}}{g_{21}}$$

$$B = \frac{g_{22}}{g_{21}} \qquad D = \frac{g_{22}}{g_{21}} - g_{12}.$$

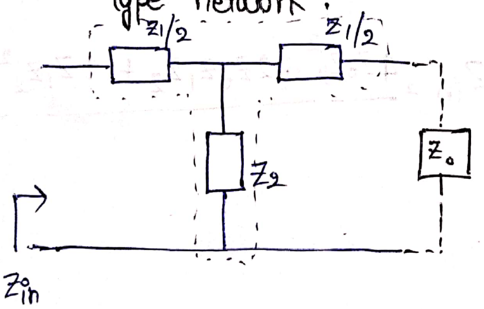
## Characteristic impedance ($Z_0$).




Defn:- In a symmetrical 2-port network, when one port is terminated with an impedance of $Z_0$ and if the measured impedance on the other port is same as the terminated impedance $Z_0$. Then it is said to be characteristic impedance.

⇒ In some cases characteristic impedance is also called as iterative impedance.

⇒ characteristic impedance for 'T' type network.




$$Z_{in} = \left[(Z_0 + Z_1/2) \,//\, Z_2\right] + \frac{Z_1}{2}$$

$$Z_{in} = \frac{\left(Z_0 + \frac{Z_1}{2}\right) \cdot Z_2}{Z_0 + \frac{Z_1}{2} + Z_2} + \frac{Z_1}{2}$$

$$= \frac{Z_0 Z_2 + \frac{Z_1 Z_2}{2}}{Z_0 + \frac{Z_1}{2} + Z_2} + \frac{Z_1}{2}$$

$$Z_{in} = \frac{2Z_0Z_2 + \frac{\cancel{2}Z_1Z_2}{\cancel{2}} + Z_0Z_1 + \frac{Z_1^2}{2} + Z_1Z_2}{2Z_0 + \frac{\cancel{2}Z_1}{\cancel{2}} + 2Z_2}$$

$$Z_{in} = \frac{2Z_0Z_2 + 2Z_1Z_2 + Z_0Z_1 + \frac{Z_1^2}{2}}{2Z_0 + Z_1 + 2Z_2}$$

if $Z_{in}$ is a characteristic impedance then $Z_{in} = Z_0$.

$$Z_0 = \frac{2Z_0Z_2 + 2Z_1Z_2 + Z_0Z_1 + \frac{Z_1^2}{2}}{2Z_0 + Z_1 + 2Z_2}$$

$$2Z_0^2 + \cancel{Z_0Z_1} + \cancel{2Z_2Z_0} = \cancel{2Z_0Z_2} + 2Z_1Z_2 + \cancel{Z_0Z_1} + \frac{Z_1^2}{2}$$

$$2Z_0^2 = 2Z_1Z_2 + \frac{Z_1^2}{2}$$

$$Z_0^2 = \frac{\cancel{2}Z_1Z_2}{\cancel{2}} + \frac{Z_1^2}{4}$$

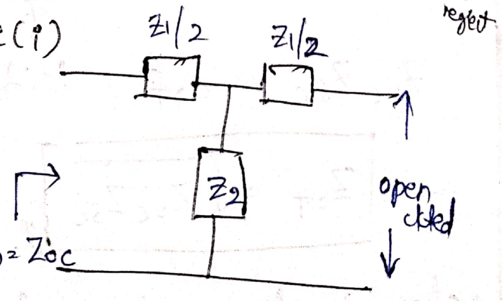
$$\boxed{Z_0 = \sqrt{Z_1Z_2 + \frac{Z_1^2}{4}}}$$

∴ Hence characteristic impedance for a 'T' type network is given by $\boxed{Z_{0T} = \sqrt{Z_1Z_2 + \frac{Z_1^2}{4}}}$ —ⓐ

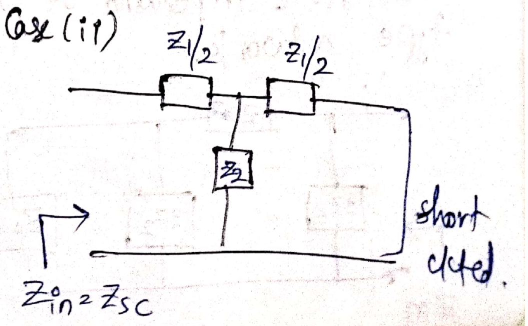
⇒ characteristic impedance for 'T' type network interms of short ckt and open ckted impedance.

O.C – Series right
S.C – parallel reject

Case (i)




Case (ii)



from case (i) (open ckted).

$$Z_{in} = Z_{oc} = \frac{Z_1}{2} + Z_2 = \frac{Z_1 + 2Z_2}{2} \quad \text{--- ①}$$

from case (2) (short ckted)

$$Z_{in} = Z_{sc} = (Z_1/2 \parallel Z_2) + \frac{Z_1}{2}$$

$$= \frac{\frac{Z_1}{2} \cdot Z_2}{\frac{Z_1}{2} + Z_2} + \frac{Z_1}{2}$$

$$Z_{sc} = \frac{\frac{2Z_1Z_2}{2} + \frac{Z_1^2}{2} + Z_1Z_2}{\frac{2Z_1}{2} + 2Z_2}$$

$$Z_{sc} = \frac{2Z_1Z_2 + \frac{Z_1^2}{2}}{Z_1 + 2Z_2} \quad \text{--- ②}$$

multiply ① * ②.

$$Z_{oc} \cdot Z_{sc} = \left(\frac{Z_1 \cancel{+ 2Z_2}}{2}\right)\left(\frac{2Z_1Z_2 + \frac{Z_1^2}{2}}{\cancel{Z_1 + 2Z_2}}\right)$$

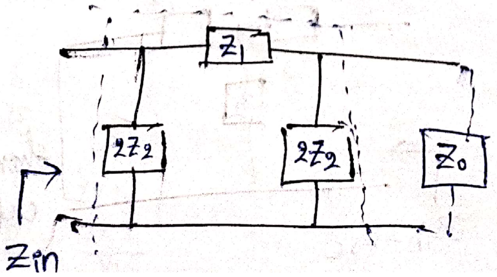
$$Z_{oc}Z_{sc} = \frac{2Z_1Z_2}{2} + \frac{Z_1^2}{4}$$

$$Z_{oc}Z_{sc} = Z_1Z_2 + \frac{Z_1^2}{4}$$

$$Z_{oc}Z_{sc} = Z_{oT}^2 \quad [\because \text{from eq}^n \text{-①}]$$

$$\boxed{Z_{oT} = \sqrt{Z_{oc}Z_{sc}}}$$

⇒ characteristic impedance of 'π' type network.




$$Z_{in} = \left[\left[Z_0 \parallel 2Z_2\right] + Z_1\right] \parallel 2Z_2$$

$$= \left[\frac{Z_0 \cdot 2Z_2}{Z_0 + 2Z_2} + Z_1\right] \parallel 2Z_2$$

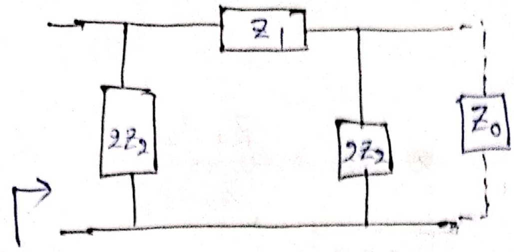
$$Z_{in} = \frac{\left[\frac{Z_0 2Z_2}{Z_0 + 2Z_2} + Z_1\right] 2Z_2}{\frac{Z_0 2Z_2}{Z_0 + 2Z_2} + Z_1 + 2Z_2}$$

$$Z_{in} = \frac{\left[\frac{Z_0 \cdot 2Z_2 + Z_1(Z_0 + 2Z_2)}{Z_0 + 2Z_2}\right] 2Z_2}{\frac{Z_0 \cdot 2Z_2 + Z_1(Z_0 + 2Z_2)}{Z_0 + 2Z_2} + 2Z_2}$$

$$Z_{in} = \frac{\left[Z_0 \cdot 2Z_2 + Z_1Z_0 + 2Z_1Z_2\right] 2Z_2}{Z_0 \cdot 2Z_2 + Z_0Z_1 + 2Z_1Z_2 + 2Z \ldots}$$

$$Z_{in} = \frac{4Z_0Z_2^2 + 2Z_0Z_1Z_2 + 4Z_1Z_2^2}{\ldots}$$

→ characteristic impedance of 'π' type network.



$Z_{in}$.

$$Z_{in} = \left[\left[Z_0 \| 2Z_2\right] + Z_1\right] \| 2Z_2 = \left[\frac{Z_0 \cdot 2Z_2}{Z_0 + 2Z_2} + Z_1\right] \| 2Z_2.$$

$$Z_{in} = \frac{\left[\dfrac{Z_0 \cdot 2Z_2}{Z_0 + 2Z_2} + Z_1\right] 2Z_2}{\left[\dfrac{Z_0 \cdot 2Z_2}{Z_0 + 2Z_2} + Z_1\right] + 2Z_2}.$$

$$Z_{in} = \frac{\left[\dfrac{Z_0 \cdot 2Z_2 + Z_1(Z_0 + 2Z_2)}{Z_0 + 2Z_2}\right] 2Z_2}{\dfrac{Z_0 \cdot 2Z_2 + Z_1(Z_0 + 2Z_2)}{Z_0 + 2Z_2} + 2Z_2}$$

$$Z_{in} = \frac{\dfrac{\left[Z_0 2Z_2 + Z_1 Z_0 + 2Z_1 Z_2\right] 2Z_2}{\cancel{Z_0 + 2Z_2}}}{\dfrac{Z_0 2Z_2 + Z_1 Z_0 + 2Z_1 Z_2 + \underbrace{2Z_2(Z_0 + 2Z_2)}_{2Z_0 Z_2 + 4Z_2^2}}{Z_0 + 2Z_2}}$$

$$Z_{in} = \frac{4Z_2^2 Z_0 + 2Z_0 Z_1 Z_2 + 4Z_2^2 Z_1}{2Z_2 Z_0 + Z_1 Z_0 + 2Z_1 Z_2 + 2Z_0 Z_2 + 4Z_2^2}$$

if $Z_{in}$ is a characteristic impedance then $Z_{in} = Z_0$.

$$Z_0 = \frac{4Z_2^2 Z_0 + 2Z_0 Z_1 Z_2 + 2Z_2^2 Z_1}{2Z_2 Z_0 + Z_1 Z_0 + 2Z_1 Z_2 + 2Z_0 Z_2 + 4Z_2^2}$$

$$2Z_2 Z_0^2 + Z_1 Z_0^2 + \cancel{2Z_0 Z_1 Z_2} + 2Z_0^2 Z_2 + \cancel{4Z_2^2 Z_0} = \cancel{4Z_2^2 Z_0} + \cancel{2Z_0 Z_1 Z_2} + 4Z_2^2 Z_1$$

$$\therefore \; 2Z_2 Z_0^2 + Z_1 Z_0^2 + 2Z_0^2 Z_2 = 4Z_2^2 Z_1$$

$$4Z_0^2 Z_2 + Z_0^2 Z_1 = 4Z_2^2 Z_1$$

$$Z_0^2 (4Z_2 + Z_1) = 4Z_1 Z_2^2$$

$$\therefore Z_0^2 = \frac{4 Z_1 Z_2^2}{4 Z_2 + Z_1} \qquad Z_{0\pi} = \sqrt{\frac{Z_1 Z_2}{1+\frac{Z_1}{4Z_2}}}$$

$$Z_0^2 = \frac{4Z_1Z_2^2}{4Z_2+Z_1} \Rightarrow Z_0 = \frac{Z_1 Z_2}{\sqrt{Z_2+Z_1}}$$

$$Z_0 = \sqrt{\frac{4Z_1Z_2^2}{4Z_2+Z_1}} = \sqrt{\frac{4Z_1Z_2^2}{4(Z_2+\frac{Z_1}{4})}}$$

$$\boxed{Z_0 = \sqrt{\frac{Z_1Z_2^2}{Z_2+\frac{Z_1}{4}}}} = Z_0 = \frac{Z_1^2 Z_2}{\sqrt{Z_2+\frac{Z_1}{4}}}$$

$$Z_0^2 = \frac{4Z_1Z_2^2}{4Z_2+Z_1} = \frac{4Z_1Z_2^2}{4Z_2\left[1+\frac{Z_1}{4Z_2}\right]}$$

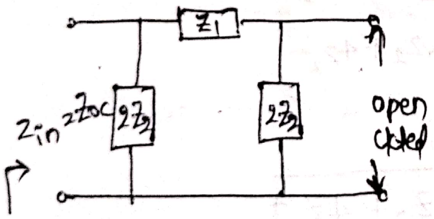
$$\boxed{Z_0 = \sqrt{\frac{Z_1Z_2^2}{1+\frac{Z_1}{4Z_2}}}} \quad \text{—(a)}$$

05/1/2024

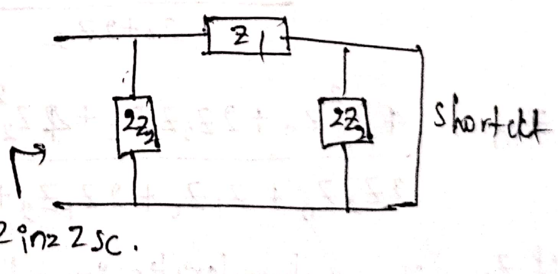
⇒ characteristic impedance of π-network interms of open ckted and short ckted impedance.

O.C → series reg<br>
S.C → parallel reg

Case (i) :-




Case (ii)




from case (i) open ckted

$$Z_{in} = Z_{oc} = [Z_1 + 2Z_2] \,||\, 2Z_2$$

$$Z_{oc} = \frac{(Z_1+2Z_2)2Z_2}{Z_1+2Z_2+2Z_2}$$

$$Z_{oc} = \frac{(Z_1+2Z_2)2Z_2}{Z_1+4Z_2} \quad \text{—(1)}$$

from case (2). [short ckted]

$$Z_{in} = Z_{sc} = 2Z_2 \| Z_1$$

$$Z_{sc} = \frac{2Z_2 Z_1}{2Z_2+Z_1} \quad \text{— (2)}$$

multiply (1) & (2)

$$Z_{oc}Z_{sc} = \left[\frac{(Z_1+2Z_2)2Z_2}{Z_1+4Z_2}\right]\left[\frac{2Z_1Z_2}{2Z_2+Z_1}\right]$$

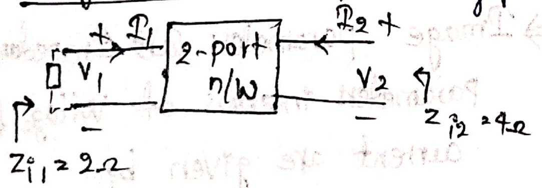
$$Z_{oc}Z_{sc} = \frac{4Z_1Z_2^2}{Z_1+4Z_2} = \frac{4Z_1Z_2^2}{4Z_2\left[1+\frac{Z_1}{4Z_2}\right]}$$

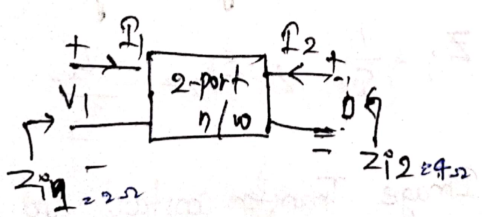
$$Z_{oc}Z_{sc} = \frac{Z_1Z_2}{1+\frac{Z_1}{4Z_2}}$$

$Z_{oc}Z_{sc} = Z_{0\pi}^2$ from eq$^n$ (a).

$$\boxed{Z_{0\pi} = \sqrt{Z_{oc}Z_{sc}}}$$ //

**Image Impedance (or) Image parameters.**




(1). If port (1) is terminated with an impedance of $Z_{i1}$ and measured impedance on 2nd port is $Z_{i2}$.

(2). If port (2) is terminated with an impedance of $Z_{i2}$ and impedance at port (1) is $Z_{i1}$.

⇒ Then $Z_{i1}$ and $Z_{i2}$ is said to image ~~imag~~ impedance (or) Impedance parameters.

w.r.t, ABCD parameters.

$$Z_{i1} = \frac{AZ_{i2}+B}{CZ_{i2}+D} \quad \text{— (1)}$$

$$Z_{i2} = \frac{B+DZ_{i1}}{A+CZ_{i1}} \quad \text{— (2)}$$

Case (i): sub (1) in (2).

$$Z_{i2} = \frac{B+D\left[\frac{AZ_{i2}+B}{CZ_{i2}+D}\right]}{A+C\left[\frac{AZ_{i2}+B}{CZ_{i2}+D}\right]}$$

$$Z_{i2} = \frac{B+\frac{ADZ_{i2}+BD}{CZ_{i2}+D}}{A+\frac{CAZ_{i2}+BC}{CZ_{i2}+D}}$$

$$Z_{i2} = \frac{\frac{BCZ_{i2}+BD+ADZ_{i2}+BD}{\cancel{CZ_{i2}+D}}}{\frac{ACZ_{i2}+AD+CAZ_{i2}+BC}{\cancel{CZ_{i2}+D}}}$$

$$Z_{i2} = \frac{BCZ_{i2}+BD+ADZ_{i2}+BD}{ACZ_{i2}+AD+CAZ_{i2}+BC}$$

$$2ACZ_{i_2}^2 + \cancel{ADZ_{i_2}} + \cancel{BCZ_{i_2}} = \cancel{BCZ_{i_2}} + 2BD + \cancel{ADZ_{i_2}}$$

$$2ACZ_{i_2}^2 = 2BD$$

$$Z_{i_2}^2 = \frac{BD}{AC}$$

$$\boxed{Z_{i_2} = \sqrt{\frac{BD}{AC}}}$$

Case (ii) sub ② in ①.

$$Z_{i_1} = \frac{A\left[\frac{B+DZ_{i_1}}{A+CZ_{i_1}}\right]+B}{C\left[\frac{B+DZ_{i_1}}{A+CZ_{i_1}}\right]+D}$$

$$Z_{i_1} = \frac{\frac{AB+ADZ_{i_1}+B}{A+CZ_{i_1}}}{\frac{CB+CDZ_{i_1}}{A+CZ_{i_1}}+D}$$

$$Z_{i_1} = \frac{AB+ADZ_{i_1}+BA+CBZ_{i_1}}{CB+CDZ_{i_1}+DA+CDZ_{i_1}}$$

$$\cancel{CBZ_{i_1}} + CDZ_{i_1}^2 + \cancel{DAZ_{i_1}} + CDZ_{i_1}^2 = AB + \cancel{ADZ_{i_1}} + BA + \cancel{CBZ_{i_1}}$$

$$2CDZ_{i_1}^2 = 2AB$$

$$Z_{i_1}^2 = \frac{AB}{CD}$$

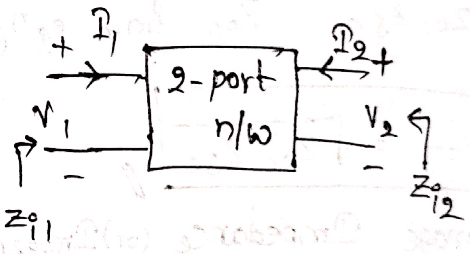
$$\boxed{Z_{i_1} = \sqrt{\frac{AB}{CD}}}$$

→ Image Transfer Constant (or) propagation constant.

Def$^n$ :- To determine any network interms of reciprocal network image Transfer constant (or) propagation constant is used.

* Determination of Assymmetrical network interms of symmetrical network is done using Image Transfer constant ($\phi$).

* Determination of Symmetrical network interms of reciprocal network is done using propagation constant ($\gamma$) gamma.




6/1/2

→ Image parameters (or) Impedance parameters interms of voltage & current are given by.

$$Z_{i_1} = \frac{V_1}{I_1}, \quad Z_{i_2} = \frac{-V_2}{I_2}$$

→ Image Transfer constant and propagation constant can be derived with the help of ABCD parameters.

→ WKT for ABCD parameters

$$V_1 = AV_2 - BI_2 \quad —①$$

$$I_1 = CV_2 - DI_2 \quad —②$$

from eqⁿ – ①

$$V_1 = AV_2 - BI_2$$

$$= V_2\left[A - B\frac{I_2}{V_2}\right]$$

$$\frac{V_1}{V_2} = A - B\frac{I_2}{V_2}$$

$$\frac{V_1}{V_2} = A - B\left(\frac{-1}{Z_{i2}}\right)$$

$$\frac{V_1}{V_2} = A + \frac{B}{Z_{i2}}$$

WKT $Z_{i2} = \sqrt{\frac{BD}{AC}}$

$$\frac{V_1}{V_2} = A + \frac{B}{\sqrt{\frac{BC}{AC}}}$$ $\cos\theta + j\sin\theta = e^{j\theta}$

$$\frac{V_1}{V_2} = A + B\sqrt{\frac{AC}{BD}} = A + \sqrt{\frac{B^2AC}{BD}}$$

$$\frac{V_1}{V_2} = A + \sqrt{\frac{ABC}{D}} \quad \text{— ③}$$

from eqⁿ – ②

$$I_1 = CV_2 - DI_2$$

$$I_1 = I_2\left[C\frac{V_2}{I_2} - D\right]$$

$$\frac{I_1}{I_2} = C\frac{V_2}{I_2} - D$$

$$= C(-Z_{i2}) - D$$

$$\frac{I_1}{I_2} = -\left[CZ_{i2} + D\right]$$

$$-\frac{I_1}{I_2} = CZ_{i2} + D$$

WKT $Z_{i2} = \sqrt{\frac{BD}{AC}}$

$$-\frac{I_1}{I_2} = C\left[\sqrt{\frac{BD}{AC}}\right] + D$$

$$-\frac{I_1}{I_2} = \sqrt{\frac{C^2BD}{AC}} + D$$

$$\frac{-I_1}{I_2} = D + \sqrt{\frac{BCD}{A}} \quad \text{— ④}$$

multiplication of ③ & ④

$$\left(\frac{V_1}{V_2}\right)\left(-\frac{I_1}{I_2}\right) = \left[A + \sqrt{\frac{ABC}{D}}\right]\left[D + \sqrt{\frac{BCD}{A}}\right]$$

$$= AD + A\sqrt{\frac{BCD}{A}} + D\sqrt{\frac{ABC}{D}} + \sqrt{\frac{ABC}{D}\cdot\frac{BCD}{A}}$$

$$= AD + \sqrt{\frac{BCDA^2}{A}} + \sqrt{\frac{ABCD^2}{D}} + \sqrt{(BC)^2}$$

$$= AD + \sqrt{ABCD} + \sqrt{ABCD} + BC$$

$$\left(\frac{V_1}{V_2}\right)\left(-\frac{I_1}{I_2}\right) = AD + BC + 2\sqrt{ABCD}$$

$$= (\sqrt{AD})^2 + (\sqrt{BC})^2 + 2\sqrt{AD}\cdot\sqrt{BC}$$

$$\left(\frac{V_1}{V_2}\right)\left(-\frac{I_1}{I_2}\right) = \left(\sqrt{AD} + \sqrt{BC}\right)^2$$

$$\sqrt{AD} + \sqrt{BC} = \sqrt{\left(\frac{V_1}{V_2}\right)\left(-\frac{I_1}{I_2}\right)}$$

WKT for ABCD parameters.

Reciprocal condition

$$AD - BC = 1$$

$$BC = AD - 1$$

$$\sqrt{AD} + \sqrt{AD-1} = \sqrt{\left(\frac{V_1}{V_2}\right)\left(\frac{-I_1}{I_2}\right)}$$

let $\sqrt{AD} = \cosh\phi$, $\sqrt{AD-1} = \sinh\phi$

$$\cosh\phi + \sinh\phi = \sqrt{\frac{V_1}{V_2}\cdot\frac{-I_1}{I_2}}$$

$$\boxed{e^{\phi} = \sqrt{\frac{V_1}{V_2}\cdot\frac{-I_1}{I_2}}}$$ Image transfer constant

WKT

$$Z_{i1} = \frac{V_1}{I_1}, \quad Z_{i2} = \frac{-V_2}{I_2}$$

$$V_1 = Z_{i1}I_1, \quad V_2 = -Z_{i2}I_2$$

Sub $V_1$ & $V_2$ in above eqⁿ.

$$e^{\phi} = \sqrt{\frac{Z_{i1}I_1}{-Z_{i2}I_2}\cdot\frac{-I_1}{I_2}}$$

$$e^{\phi} = \sqrt{\frac{Z_{i1}}{Z_{i2}}\left(\frac{I_1}{I_2}\right)^2} = \frac{\sqrt{Z_{i1}}}{\sqrt{Z_{i2}}}\left(\frac{I_1}{I_2}\right)^2$$

$$\therefore \boxed{e^{\phi} = \frac{I_1}{I_2}\sqrt{\frac{Z_{i1}}{Z_{i2}}}}$$ Image transfer ⑤ constant

for symmetrical network

$$Z_{i1} = Z_{i2}$$

from eq^n ⑤

$$e^{\phi} = \frac{I_1}{I_2}\sqrt{\frac{Z_{i1}}{Z_{i1}}}$$

$$\therefore \boxed{e^{\phi} = \frac{I_1}{I_2} = \gamma}$$ propagation constant.

$$\therefore \boxed{\phi = \ln\left(\frac{I_1}{I_2}\right) = \gamma}$$

$$\boxed{\gamma = \alpha + j\beta}$$

$\alpha \rightarrow$ attenuation

$\beta \rightarrow$ phase shift.

08/1/2023.

## Network Functions :-

Def^n :- A function ~~whe~~ with S-domain that can describe a network is called as Network function.

⇒ Network functions are of 2 types:-

1) Driving point function.
2) Transfer function.

**Driving point function** :- It is the ratio of variables at same port.

⇒ There exists 4 types of driving point functions.

1) Input impedance driving point function ($Z_{11}(s)$)

Def^n :- It is the ratio of voltage at port ① to current at port ① in s domain.

$$\boxed{Z_{11}(s) = \frac{V_1(s)}{I_1(s)}}$$

2) Input admittance driving point function. ($Y_{11}(s)$).

Def^n :- It is the ratio of current at port ① to voltage at port ①, in s-domain.

$$\boxed{Y_{11}(s) = \frac{I_1(s)}{V_1(s)}}$$

3) output Impedance driving point function ($Z_{22}(s)$).

It is the ratio of voltage at port ② to current at port ② in s-domain

$$\boxed{Z_{22}(s) = \frac{V_2(s)}{I_2(s)}}$$

4) output admittance driving point function ($Y_{22}(s)$)

It is the ratio of current at port 2 to voltage at port 2 in s-domain

$$\boxed{Y_{22}(s) = \frac{I_2(s)}{V_2(s)}}$$

## Transfer function :-

Def^n :- It is ratio of variable at different ports.

⇒ There exists 4 types of Transfer functions.

1) Voltage Transfer function:- It is the ratio of voltage transfer at one port to voltage transfer at other port.

$$\alpha_{12}(s) = \frac{V_1(s)}{V_2(s)}$$

$$\alpha_{21}(s) = \frac{V_2(s)}{V_1(s)}.$$

2) Current Transfer function :-

→ It is the ratio of current transfer at one port to current transfer at other port.

$$G_{12}(s) = \frac{I_1(s)}{I_2(s)}$$

$$G_{21}(s) = \frac{I_2(s)}{I_1(s)}$$

3) Impedance Transfer function.

→ It is the ratio of voltage transfer at one port to current transfer at other part.

$$Z_{12}(s) = \frac{V_1(s)}{I_2(s)}, \quad Z_{21}(s) = \frac{V_2(s)}{I_1(s)}$$

4) Admittance Transfer function.

It is the ratio of current transfer at one port to voltage transfer at other port.

$$Y_{12}(s) = \frac{I_1(s)}{V_2(s)}, \quad Y_{21}(s) = \frac{I_2(s)}{V_1(s)}$$

## Poles & zeros :-

let us consider a transfer function H(s) as a ratio of 2 different polynomials P(s) & Q(s).

$$\therefore H(s) = \frac{P(s)}{Q(s)}$$

→ The polynomial P(s) has 'm' no. of factors.

similarly the polynominal Q(s) has 'n' no. of factors.

→ Both the polynomials in the form of m factors and n factors, are represented as

$$\boxed{\therefore H(s) = \frac{k(s-z_1)(s-z_2)\cdots(s-z_m)}{(s-P_1)(s-P_2)\cdots(s-P_n)}}$$

poles :- Roots (or) the factors of denominator polynomials, are called as Poles.

→ poles are always represented with 'X' → cross mark

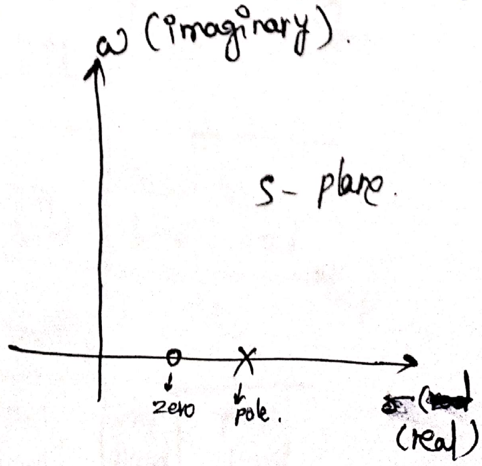
Zeros :- Roots (or) the factors of numerator polynominals, are called as zeros.

→ zeros are always represented with 'O' → circle

** poles and zeros are used to determine stability of a network (or) system.

→ poles and zeros are represented in S-plane.

→ where S-plane is, the graphical representation of real & imaginary paths.



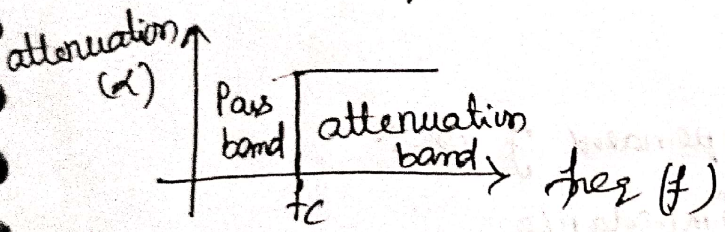
# Unit - 4

## Filters & attenuators

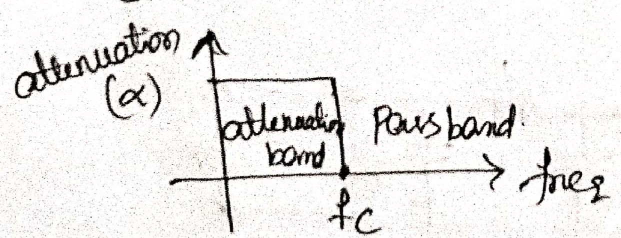
**filters :** A n/w which allows particular band of frequencies to pass through it by suppressing other band of frequencies is called "filter".

### Classification of filters :

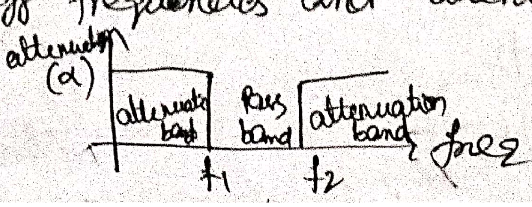
① **Low pass filter :** A filter which allows frequencies lower than cut-off freq ($f_c$) and attenuates frequencies higher than cut-off freq is called "LPF".




② **high pass filter :** A filter that allows freq higher than cut-off freq ($f_c$) and attenuates freq lower than cut-off freq is called "HPF".



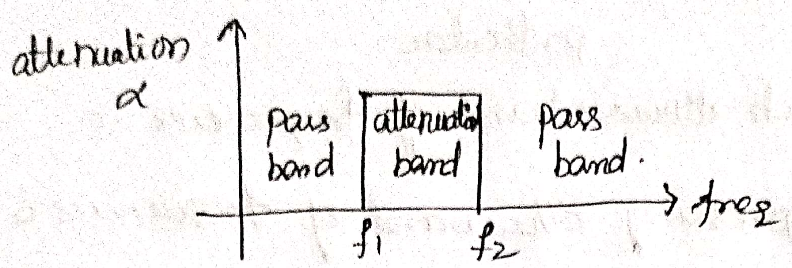

③ **Band pass filter :** A filter that ~~attenuates~~ allows frequencies b/w two cut-off frequencies and attenuates all other freqs is called "BPF"




$f_1 \rightarrow$ lower cut off freq

$f_2 \rightarrow$ upper cut-off freq

(4) **Band Elimination filter (or) Band Reject filter :-** A filter which allows outside freq of two cut-off freqs $f_1$ & $f_2$ and ~~rep~~ attenuates freq b/w $f_1$ & $f_2$ is called "BRF (or) BEF".




**Cut-off freq :** Frequency which seperates pass band and attenuation band is called cut-off freq.

**Constant K filter (prototype filter) :** In this filter the series & shunt impedances ($Z_1$ & $Z_2$) are such that,

→ shunt of impedance

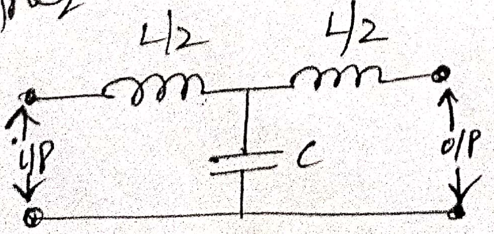
$$Z_1 \cdot Z_2 = R_0^2 = K \text{ (Const)}.$$

series arm impedance

$R_0 \rightarrow$ Real number & independent of freq.

↳ is called design impedance

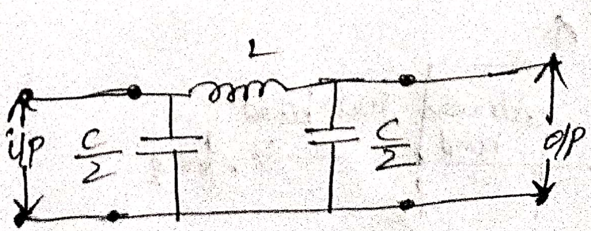
**Constant K low Pass filter :** simplest type of filter which allows lower freq upto cut off freq and attenuates higher freq.




'T' – Config LPF

→ series arm is L/2

→ Shunt arm is C




'π' – Configuration LPF

→ Series arm is L

→ shunt arm is C/2

Total series impedance

$$Z_1 = j\omega L \rightarrow (1)$$

total shunt impedance

$$Z_2 = \frac{1}{j\omega C} = \frac{-j}{\omega C} \rightarrow (2)$$

multiply (1) × (2)

$$Z_1 \cdot Z_2 = j\omega L \cdot \frac{1}{j\omega C} = \frac{L}{C}$$

$R_0$ (or) K

→ design impedance

Let $\frac{L}{C} = R_0^2 \rightarrow (3)$ ($\because \frac{L}{C}$ is real quantity)

$= K^2$

'T' N/w

WKT

$$Z_{0T} = \sqrt{Z_1 Z_2 + \frac{Z_1^2}{4}}$$

$$Z_{0T} = \sqrt{\frac{L}{C} + \frac{(j\omega L)^2}{4}}$$

$$= \sqrt{\frac{L}{C} + \frac{\omega^2 L^2}{4}}$$

$$= \sqrt{\frac{L}{C}\left[1 - \frac{\omega^2 LC}{4}\right]}$$

$$= \sqrt{\frac{L}{C}} \cdot \sqrt{1 - \frac{\omega^2 LC}{4}}$$

$$\boxed{Z_{0T} = R_0 \sqrt{1 - \frac{\omega^2 LC}{4}}} \rightarrow (4)$$

($R_0$ or K)

for characteristic impedance

$$Z_{0T} = R_0 \sqrt{1 - \frac{\omega^2}{4/LC}}$$

Let $\omega_c^2 = \frac{4}{LC}$

$$Z_{0T} = R_0 \sqrt{1 - \frac{\omega^2}{\omega_c^2}}$$

$$Z_{0T} = R_0 \sqrt{1 - \frac{(2\pi f)^2}{(2\pi f_c)^2}}$$

$$\boxed{Z_{0T} = R_0 \sqrt{1 - \left(\frac{f}{f_c}\right)^2}} \rightarrow (5)$$

($R_0$ or K)

from eqn (4)

if $\frac{\omega^2 LC}{4} < 1$, $Z_{OT}$ is real

$\frac{\omega^2 LC}{4} > 1$, $Z_{OT}$ is imaginary.

∴ * → $Z_{OT}$ is characteristic impedance of pass band when $\frac{\omega^2 LC}{4} < 1$ i.e $Z_{OT}$ is real

** → $Z_{OT}$ is Characteristic impedance of attenuation band when $\frac{\omega^2 LC}{4} > 1$ i.e $Z_{OT}$ is imaginary

at one particular condition

$$\frac{\omega^2 LC}{4} = 1$$

$$\omega^2 LC = 4$$

$$\omega^2 = \frac{4}{LC}$$

$$\omega = \frac{2}{\sqrt{LC}}$$

$$\cancel{2}\pi f = \frac{\cancel{2}}{\sqrt{LC}}$$

$$f = \frac{1}{\pi\sqrt{LC}}$$

at $f = f_c$

$$\boxed{f_c = \frac{1}{\pi\sqrt{LC}}} \quad \text{IIIly} \quad \boxed{\omega_c = \frac{2}{\sqrt{LC}}}$$

cut off freq      angular freq.

for π-n/w Characteristic Impedance:

WKT $Z_{0\pi} = \frac{Z_1 Z_2}{Z_{0T}}$

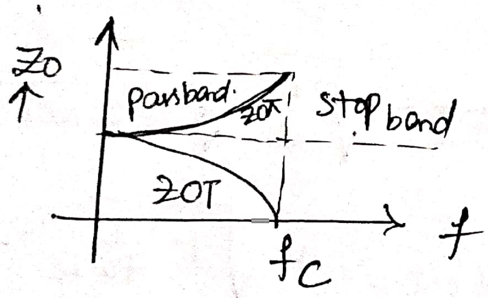
$= \frac{R_0^2}{R_0\sqrt{1-\left(\frac{f}{f_c}\right)^2}}$ [∵ from (3) & (5)]

$$\boxed{Z_{0\pi} = \frac{R_0}{\sqrt{1-\left(\frac{f}{f_c}\right)^2}}} \rightarrow (6)$$

from (6)

⇒ if $f < f_c$, $Z_{0\pi}$ is real for pass band.

if $f > f_c$, $Z_{0\pi}$ is imaginary




*** for a Constant K - LPF

$K = \sqrt{\frac{L}{C}}$

$L = \frac{K}{\pi f_c}$

$C = \frac{1}{\pi f_c K}$.

Attenuation (α) & phase shift (β) of LPF:

propagation Constant $\cosh\gamma = 1 + \frac{Z_1}{2Z_2}$

where

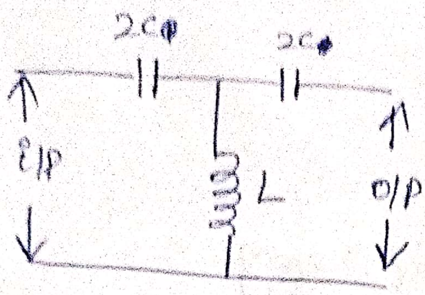
$\gamma = \alpha + j\beta$ (α → attenuation, β → phase Const)

~~In filters $Z_1 = jX_1$ & $Z_2 = jX_2$ where~~

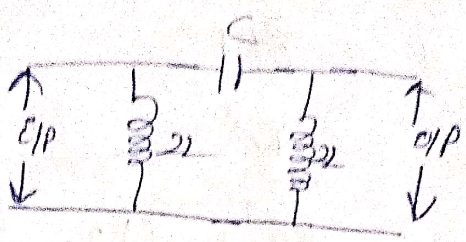
In pass band : $\alpha = 0$, $\beta = 2\sin^{-1}\left(\frac{f}{f_c}\right)$ for $f < f_c$

In attenuation band : $\alpha = 2\cosh^{-1}\left(\frac{f}{f_c}\right)$ ; $\beta = \pi$ for $f > f_c$

## constant K- high pass filter / prototype High pass filter :




'T' - section HPF




'π' - section HPF

Here $Z_1 = \frac{1}{j\omega C} \rightarrow ①$

$Z_2 = j\omega L \rightarrow ②$

$Z_1 \cdot Z_2 = \frac{1}{j\omega C} \cdot j\omega L = \frac{L}{C} = K^2$

~~$Z_1 Z_2$~~ $\boxed{K = \sqrt{\frac{L}{C}}} \rightarrow ③$ ; $Z_1 Z_2 = \frac{L}{C}$

WKT for 'T' section

$Z_{OT} = \sqrt{Z_1 Z_2 + \frac{Z_1^2}{4}}$

$= \sqrt{\frac{L}{C} + \frac{\left(\frac{1}{j\omega C}\right)^2}{4}}$

$= \sqrt{\frac{L}{C} + \frac{1}{4 j^2 \omega^2 C^2}}$

$Z_{OT} = \sqrt{\frac{L}{C} - \frac{1}{4\omega^2 C^2}}$

$Z_{OT} = \sqrt{\frac{L}{C}\left[1 - \frac{1}{4\omega^2 CL}\right]}$

$Z_{OT} = \sqrt{\frac{L}{C}} \sqrt{1 - \frac{1}{4\omega^2 LC}}$

$Z_{OT} = K \sqrt{1 - \frac{1}{4\omega^2 LC}} \rightarrow ④$

→ if $4\omega^2 LC > 1$, $Z_{OT}$ is real and filter works in pass band

→ if $4\omega^2 LC < 1$, $Z_{OT}$ is imaginary filter operates in attenuation band

Cut-off freq is obtained

for $4\omega^2 LC = 1$

$4\omega_c^2 LC = 1$

$\omega_c^2 = \frac{1}{4LC}$

$\omega_c = \sqrt{\frac{1}{4LC}}$

$$\boxed{\omega_c = \frac{1}{2\sqrt{LC}}} \quad \text{— } (5)$$

$2\pi f_c = \frac{1}{2\sqrt{LC}}$

$$\boxed{f_c = \frac{1}{4\pi\sqrt{LC}}}$$

from (4)

~~$Z_{0T} = k\sqrt{\frac{1}{2\sqrt{LC}} \cdot \frac{1}{2\pi}}$~~

~~WKT $\omega = 2\pi f$~~

$Z_{0T} = k\sqrt{1 - \frac{1}{4LC} \cdot \frac{1}{\omega^2}}$

$= k\sqrt{1 - \frac{1}{\omega_c^2}}$

$Z_{0T} = k\sqrt{1 - \frac{\omega_c^2}{\omega^2}}$ $\left[\because \text{from (5)} \; \omega_c^2 = \frac{1}{4LC}\right]$

$= k\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}$

$$\boxed{Z_{0T} = k\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

WKT for 'π' n/w

$Z_{0\pi} = \frac{Z_1 Z_2}{Z_{0T}}$

$= \frac{k^2}{k\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$

$$\boxed{Z_{0\pi} = \frac{k}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}}$$

<u>Attenuation ($\alpha$)</u> & <u>phase shift ($\beta$)</u> <u>of HPF:</u>

* for 'T' n/w propagation constant '$\gamma$' is

$\cosh\gamma = 1 + \frac{Z_1}{2Z_2}$

WKT $\gamma = \alpha + j\beta$

from n/w

$Z_1 = \frac{1}{j\omega C}$

$Z_2 = j\omega L$

$$\cosh(\alpha+j\beta) = 1 + \frac{\frac{1}{j\omega C}}{2 j\omega L}$$

$$\cosh(\alpha+j\beta) = 1 + \frac{1}{2j^2\omega^2 LC}$$

$$\cosh(\alpha+j\beta) = 1 - \frac{1}{2\omega^2 LC}$$

$$\cosh\alpha \cdot \cosh j\beta + \sinh\alpha \cdot \sinh j\beta = 1 - \frac{1}{2\omega^2 LC}$$

but $\cosh j\beta = \cos\beta$

$\sinh j\beta = j\sin\beta$

$$\cosh\alpha \cdot \cos\beta + \sinh\alpha \cdot j\sinh\beta = 1 - \frac{1}{2\omega^2 LC}$$

Compare real and imaginary terms.

$$\cosh\alpha \cdot \cos\beta = 1 - \frac{1}{2\omega^2 LC} \quad (a) \qquad \sinh\alpha \sinh\beta = 0$$

For passband: $\alpha = 0$, $\beta = 2\sin^{-1}\left(\frac{\omega_c}{\omega}\right)$

from (a)

$$\cos 0 \cos\beta = 1 - \frac{1}{2\omega^2 LC}$$

$$\cos\beta = 1 - \frac{1}{2\omega^2 LC}$$

WKT $\cos\beta$ lies b/w $-1$ to $1$

if $\cos\beta = 1$

$$1 = 1 - \frac{1}{2\omega^2 LC}$$

$$\frac{1}{2\omega^2 LC} = 0$$

$$\boxed{\omega = \infty \ \& \ f = \infty}$$

$(\because \frac{1}{\infty} = 0)$ $\therefore \omega = \infty$

if $\cos\beta = -1$

$$-1 = 1 - \frac{1}{2\omega^2 LC}$$

$$\frac{1}{2\omega^2 LC} = 2$$

$$4\omega^2 LC = 1$$

$$\omega^2 = \frac{1}{4LC}$$

Let $\omega = \omega_c$.

$$\omega_c = \frac{1}{2\sqrt{LC}}$$

$$f_c = \frac{1}{4\pi\sqrt{LC}}$$

for attenuation band : $\beta = n\pi$, $\alpha = \cosh^{-1}\left[2\left(\frac{\omega_c}{\omega}\right)^2 - 1\right]$

## Design parameters :

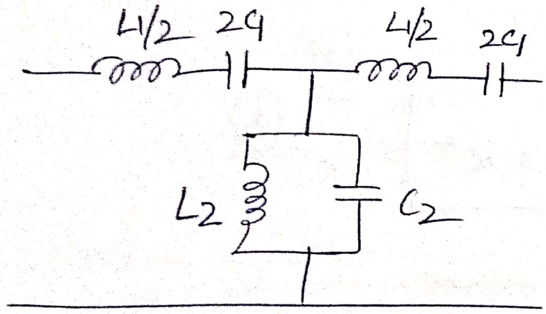
$$K = \sqrt{\frac{L}{C}}$$

$$f_c = \frac{1}{4\pi\sqrt{LC}}$$

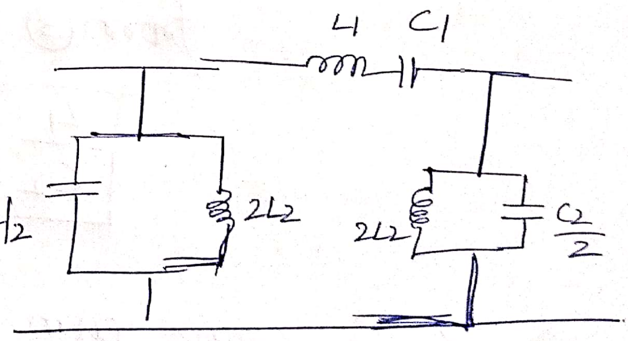
$$L = \frac{K}{4\pi f_c}$$

$$C = \frac{1}{4\pi K f_c}$$

## Design of Constant K - bandpass filter :-




T - section.




π - sec.

→ each arm is a resonant ckt

→ each arm in the above ckt has same resonant freq i.e.

→ resonant freq of shunt arm and series arm are made equal to obtain characteristics of BPF.

for equal resonant freq

$$X_L = X_C$$

$$\omega L = \frac{1}{\omega C}$$

for series arm $\quad \omega_0 \cdot \frac{L_1}{2} = \frac{1}{\omega_0 \cdot 2C_1}$

$$\omega_0^2 L_1 C_1 = 1 \rightarrow (1)$$

for shunt arm $\omega_0 L_2 = \frac{1}{\omega_0 C_2}$

$$\omega_0^2 L_2 C_2 = 1 \rightarrow (2)$$

Under resonance $(1) = (2)$

$$\cancel{\omega_0^2} L_1 C_1 = \cancel{\omega_0^2} L_2 C_2$$

$$L_1 C_1 = L_2 C_2 \rightarrow (3)$$

LDKT $K = \sqrt{\frac{L}{C}}$

$$K^2 = \frac{L}{C}$$

$\therefore$ from (3)

$$\boxed{\frac{L_1}{C_2} = \frac{L_2}{C_1} = K^2} \quad (a)$$

$\Rightarrow$ $Z_1$ is series arm

$\Rightarrow$ $Z_2$ is shunt arm

for $Z_1$:

$$Z_1 = j\omega L_1 + \frac{1}{j\omega C_1}$$

$$Z_1 = j\left[\omega L_1 + \frac{1}{j^2 \omega C_1}\right]$$

$$Z_1 = j\left[\omega L_1 - \frac{1}{\omega C_1}\right]$$

$$Z_1 = j\left[\frac{\omega^2 L_1 C_1 - 1}{\omega C_1}\right] \rightarrow (4)$$

for $\underline{Z_2}$ : $L_2 \parallel C_2$

$$= j\omega L_2 \parallel \frac{1}{j\omega C_2}$$

$$Z_2 = \frac{j\omega L_2 \cdot \frac{1}{j\omega C_2}}{j\omega L_2 + \frac{1}{j\omega C_2}}$$

$$Z_2 = \frac{\frac{L_2}{C_2}}{\frac{j^2\omega^2 L_2 C_2 + 1}{j\omega C_2}}$$

$$Z_2 = \frac{j\omega L_2 \cancel{C_2}}{\cancel{C_2}\left[-\omega^2 L_2 C_2 + 1\right]}$$

$$Z_2 = \frac{j\omega L_2}{\left[1 - \omega^2 L_2 C_2\right]} \longrightarrow (5)$$

$$Z_1 \cdot Z_2 = \frac{j}{\cancel{\omega} C_1}\left[\omega^2 L_1 C_1 - 1\right]\left[\frac{j\cancel{\omega} L_2}{1 - \omega^2 L_2 C_2}\right]$$

$$Z_1 Z_2 = j^2 \cdot \frac{L_2}{C_1}\left[\frac{\omega^2 L_1 C_1 - 1}{1 - \omega^2 L_2 C_2}\right]$$

$$Z_1 Z_2 = \frac{L_2}{C_1}\left[\frac{1 - \omega^2 L_1 C_1}{1 - \omega^2 L_2 C_2}\right]$$

from eqn (3) $L_1 C_1 = L_2 C_2$

$$Z_1 Z_2 = \frac{L_2}{C_1}\left[\frac{1 - \omega^2 L_1 C_1}{1 - \omega^2 L_1 C_1}\right]$$

$$Z_1 Z_2 = \frac{L_2}{C_1}$$

$$\boxed{Z_1 Z_2 = K^2} \quad [\because \text{from } (a)]$$

$\rightarrow (6)$

→ As freq of series arm and shunt arm is made equal then

Value of $Z_1$ at lower cut-off freq $\omega_1$ = $-$ [Value of $Z_1$ at higher cut off freq $\omega_2$]

$$Z_1 = j\omega L_1 + \frac{1}{j\omega C_1}$$

$$Z_1 \text{ at } \omega_1 = -[Z_1 \text{ at } \omega_2]$$

$$j\omega_1 L_1 + \frac{1}{j\omega_1 C_1} = -\left[j\omega_2 L_1 + \frac{1}{j\omega_2 C_1}\right]$$

$$\cancel{j}\left[\omega_1 L_1 + \frac{1}{j^2\omega_1 C_1}\right] = -\cancel{j}\left[\omega_2 L_1 + \frac{1}{j^2\omega_2 C_1}\right]$$

$$\omega_1 L_1 - \frac{1}{\omega_1 C_1} = -\left[\omega_2 L_1 - \frac{1}{\omega_2 C_1}\right]$$

$$\omega_1 L_1 - \frac{1}{\omega_1 C_1} = \frac{1}{\omega_2 C_1} - \omega_2 L_1$$

$$\frac{\omega_1^2 L_1 C_1 - 1}{\omega_1 \cancel{C_1}} = \frac{1 - \omega_2^2 L_1 C_1}{\omega_2 \cancel{C_1}}$$

from ① $L_1 C_1 = \frac{1}{\omega_0^2}$

$$\frac{\frac{\omega_1^2}{\omega_0^2} - 1}{\omega_1} = \frac{1 - \omega_2^2 \cdot \frac{1}{\omega_0^2}}{\omega_2}$$

$$\left[\frac{\omega_1^2}{\omega_0^2} - 1\right] = \frac{\omega_1}{\omega_2}\left[1 - \frac{\omega_2^2}{\omega_0^2}\right]$$

$$\left[\frac{\omega_1^2 - \omega_0^2}{\omega_0^2}\right] = \frac{\omega_1}{\omega_2}\left[\frac{\omega_0^2 - \omega_2^2}{\omega_0^2}\right].$$

$$\omega_2(\omega_1^2 - \omega_0^2) = \omega_1(\omega_0^2 - \omega_2^2)$$

$$\omega_2\omega_1^2 - \omega_2\omega_0^2 = \omega_1\omega_0^2 - \omega_1\omega_2^2$$

$$\omega_2\omega_1^2 + \omega_1\omega_2^2 = \omega_1\omega_0^2 + \omega_2\omega_0^2$$

$$\omega_1\omega_2\cancel{(\omega_1 + \omega_2)} = \omega_0^2\cancel{(\omega_1 + \omega_2)}$$

$$\boxed{\begin{aligned} \omega_0^2 &= \omega_1\omega_2 \\ f_0^2 &= f_1 f_2 \\ f_0 &= \sqrt{f_1 f_2} \end{aligned}}$$

Cut-off freq for Konstant K-type band pass filter.

## Design parameters

$L_1 C_1 = L_2 C_2$

→ $K = \sqrt{\frac{L_1}{C_2}} = \sqrt{\frac{L_2}{C_1}}$

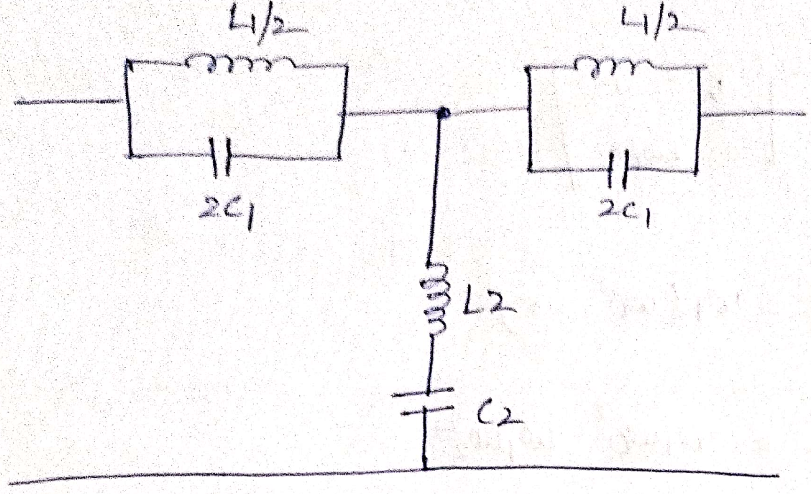
→ $L_1 = \frac{K}{\pi(f_2 - f_1)}$

→ $C_2 = \frac{1}{K\pi(f_2 - f_1)}$

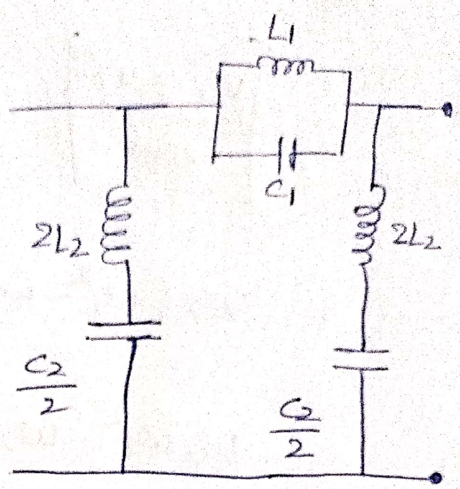
→ $L_2 = \frac{K(f_2 - f_1)}{4\pi f_1 f_2}$

→ $C_1 = \frac{(f_2 - f_1)}{4\pi K f_1 f_2}$

Constant K-type Band stop filter : Allows freq lower than $f_1$ & greater than $f_2$. [LPF with HPF]




T-Section.




$Z_1 \rightarrow$ parallel tuned ckt

$Z_2 \rightarrow$ series tuned ckt.

→ Resonant freq of shunt arm and series arm are made equal to get characteristics of BSF.

$$X_L = X_C$$

$$\omega L = \frac{1}{\omega C}$$

for series arm $\quad \omega_0 \cdot \frac{L_1}{2} = \frac{1}{\omega_0 2C_1} \Rightarrow \omega_0^2 L_1 C_1 = 1$

for shunt arm $\quad \omega_0 \cdot L_2 = \frac{1}{\omega_0 \cdot C_2} \Rightarrow \omega_0^2 L_2 C_2 = 1$

Under resonance

$$\omega_0^2 L_1 C_1 = \omega_0^2 L_2 C_2$$

$$L_1 C_1 = L_2 C_2 \rightarrow (a)$$

WKT $K^2 = \frac{L}{C}$

$$\therefore \boxed{\frac{L_1}{C_2} = \frac{L_2}{C_1} = k^2}$$

$Z_1 = (L_1, C_1)$ parallel

$$Z_1 = j\omega L_1 \parallel \frac{1}{j\omega C_1}$$

$$= \frac{j\omega L_1 \cdot \frac{1}{j\omega C_1}}{j\omega L_1 + \frac{1}{j\omega C_1}}$$

$$= \frac{L_1 \cdot j\omega \cancel{C_1}}{\cancel{C_1}\left[j^2\omega^2 L_1 C_1 + 1\right]}$$

$$Z_1 = \frac{j\omega L_1}{1-\omega^2 L_1 C_1}$$

$$Z_2 = j\omega L_2 + \frac{1}{j\omega C_2}$$

$$= \frac{j^2\omega^2 L_2 C_2 + 1}{j\omega C_2}$$

$$= \frac{1-\omega^2 L_2 C_2}{j\omega C_2}$$

(a)

$$Z_1 \cdot Z_2 = \left[\frac{j\omega L_1}{1-\omega^2 L_1 C_1}\right]\left[\frac{1-\omega^2 L_2 C_2}{j\omega C_2}\right]$$

$$= \frac{L_1}{C_2}\left[\frac{1-\omega^2 L_2 C_2}{1-\omega^2 L_1 C_1}\right]$$

from (a) $L_1 C_1 = L_2 C_2$

i.e. $1-\omega^2 L_1 C_1 = 1-\omega^2 L_2 C_2$

$$\therefore Z_1 Z_2 = \frac{L_1}{C_2}\left[\frac{1-\omega^2 L_2 C_2}{1-\omega^2 L_2 C_2}\right]$$

$$Z_1 Z_2 = \frac{L_1}{C_2}$$

$$\boxed{Z_1 Z_2 = k^2}$$

→ cut off freq of k-type band stop filter is same as BPF i.e

$$\boxed{f_0 = \sqrt{f_1 f_2}}$$

Design parameters:

→ $\frac{L_1}{C_2} = \frac{L_2}{C_1} = k^2$

$$L_1 = \frac{k(f_2 - f_1)}{\pi f_1 f_2}$$

$$C_2 = \frac{(f_2 - f_1)}{k \pi f_1 f_2}$$

$$L_2 = \frac{k}{4\pi (f_2 - f_1)}$$

$$C_1 = \frac{1}{4\pi k (f_2 - f_1)}$$

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(P1) Design a band stop / Elimination filter having design impedance of 600Ω and cut-off freq are 2KHz & 6KHz.

Sol: BSF

$K = 600\Omega$

$f_1 = 2KHz$

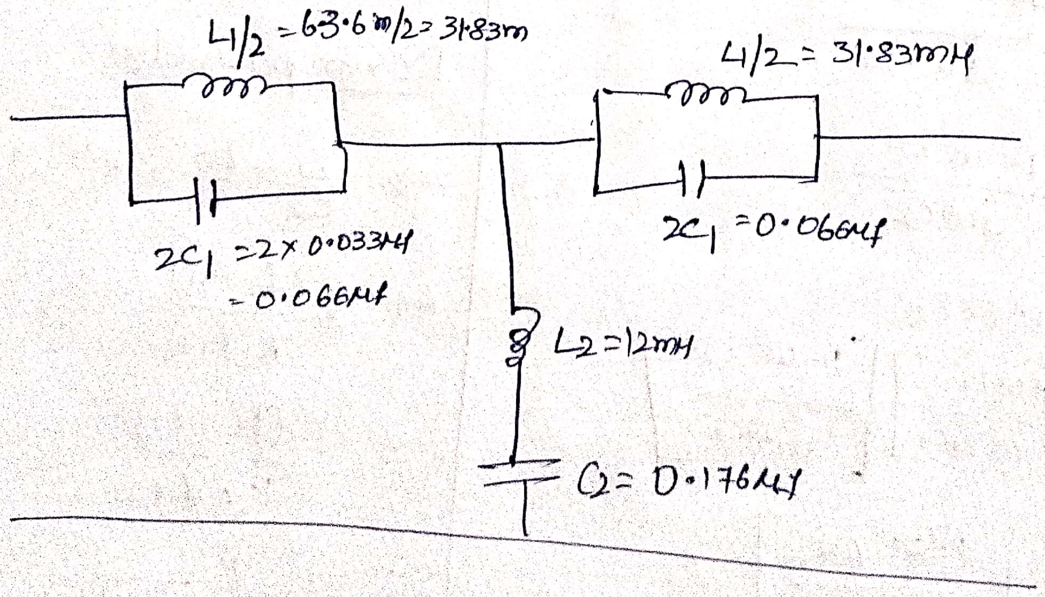
$f_2 = 6KHz$

① $L_1 = \dfrac{K(f_2 - f_1)}{\pi f_1 \cdot f_2} = \dfrac{600 \times (6-2) \times 10^3}{\pi \times 2 \times 10^3 \times 6 \times 10^3} = 63.6 \, mH$

② $C_2 = \dfrac{f_2 - f_1}{K \pi f_1 f_2} = \dfrac{(6-2) \times 10^3}{600 \times \pi \times 2 \times 6 \times 10^3 \times 10^3} = 0.176 \mu F$

③ $L_2 = \dfrac{K}{4\pi(f_2 - f_1)} \quad \dfrac{600}{4\pi(6-2) \times 10^3} = 12 mH$

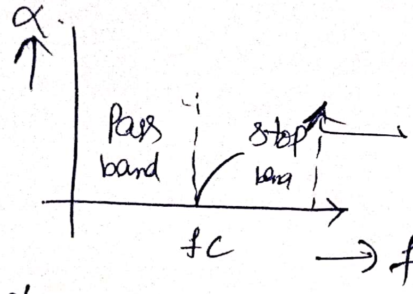
④ $C_1 = \dfrac{1}{4\pi K (f_2 - f_1)} = \dfrac{1}{4\pi \times 600 (6-2) \times 10^3} = 0.033 \mu f$



## Limitations of Constant K-type filters:

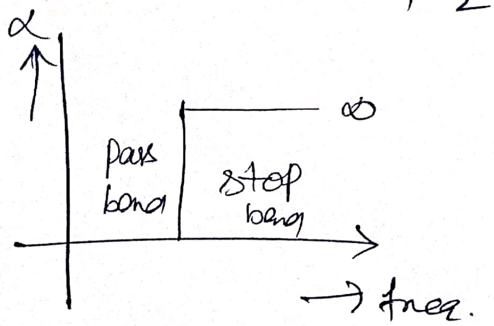
→ Characteristic impedance of the filter ckt doesn't remain constant over the pass band and it is a function of freq. i.e., it varies with freq.

→ Its attenuation doesn't rise abruptly beyond the cut-off freq as shown for LPF




$\alpha = 2\cosh^{-1}\left(\frac{f}{f_c}\right)$

(actual case)




ideally $\alpha = \infty$

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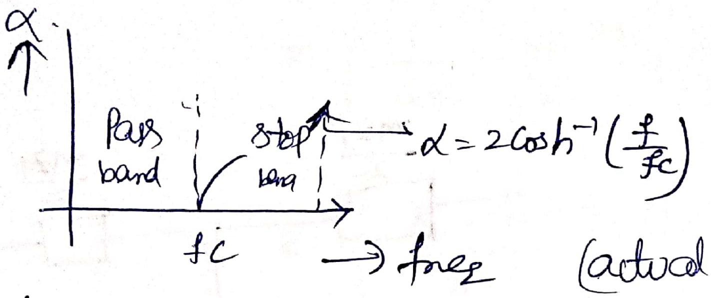
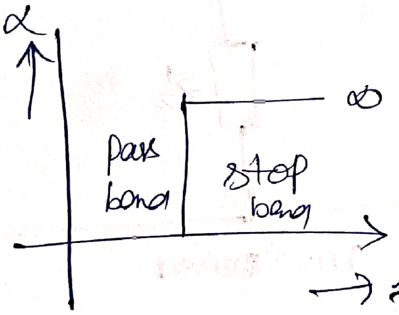



(actual case)




ideally $\alpha = \infty$

## m - Derived filters:

These are designed from Constant K-type filters

To obtain m-derived filter from Constant K-type filter

→ series impedance is multiplied by factor 'm'

→ Shunt " " divided by factor 'm'

→ An addition impedance is added either in series or in parallel.

→ the resonant freq of shunt arm must be higher than that of constant K-type filter.

$$f_\infty > f_c$$
(resonant fre)

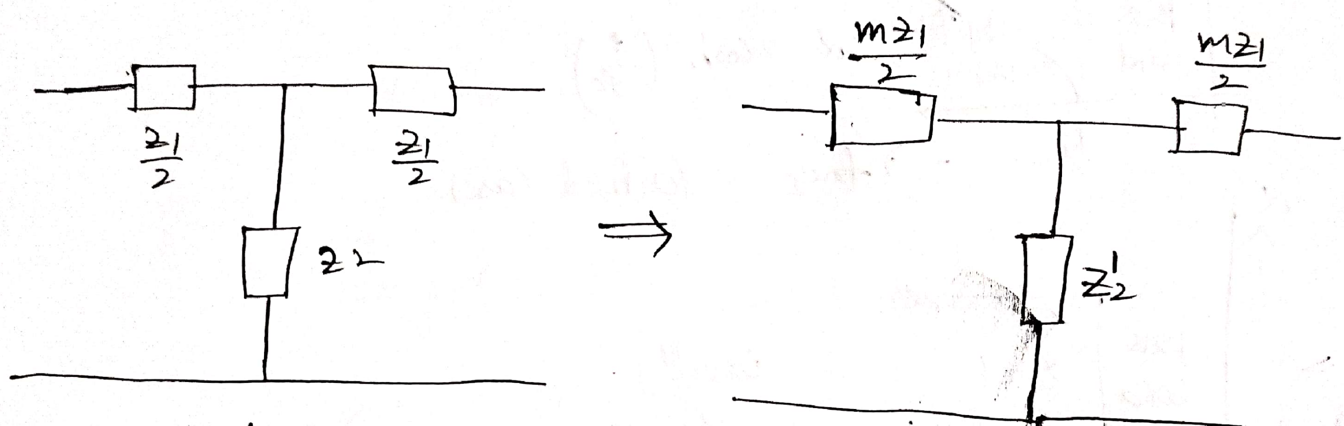
The expression for attenuation is

$$\alpha = 2\cosh^{-1}\sqrt{\left|\frac{Z_1}{4Z_2}\right|}$$

at $f = f_\infty$, $Z_2 = 0$

$\therefore \alpha = \infty$, This produces sharp cut-off (or) attenuation.

## m - Derived T-section :




Constant K- type.

m-derived

→ $Z_2'$ will have same characteristic impedance as that of $Z_2$

$$Z_{OT} = Z_{OT}'$$

for m-derived

~~$Z_1 = Z_1/2$~~

~~$Z_2 = Z_2'$~~

$$\sqrt{Z_1Z_2 + \frac{Z_1^2}{4}} = \sqrt{\frac{(mZ_1)^2}{4} + mZ_1Z_2'}$$

$$\cancel{\sqrt{}}\; Z_1Z_2 + \frac{Z_1^2}{4} = \cancel{\sqrt{}}\; mZ_1Z_2' + \frac{m^2Z_1^2}{4}$$

$$Z_1Z_2 + \frac{Z_1^2}{4} = mZ_1Z_2' + \frac{m^2Z_1^2}{4}$$

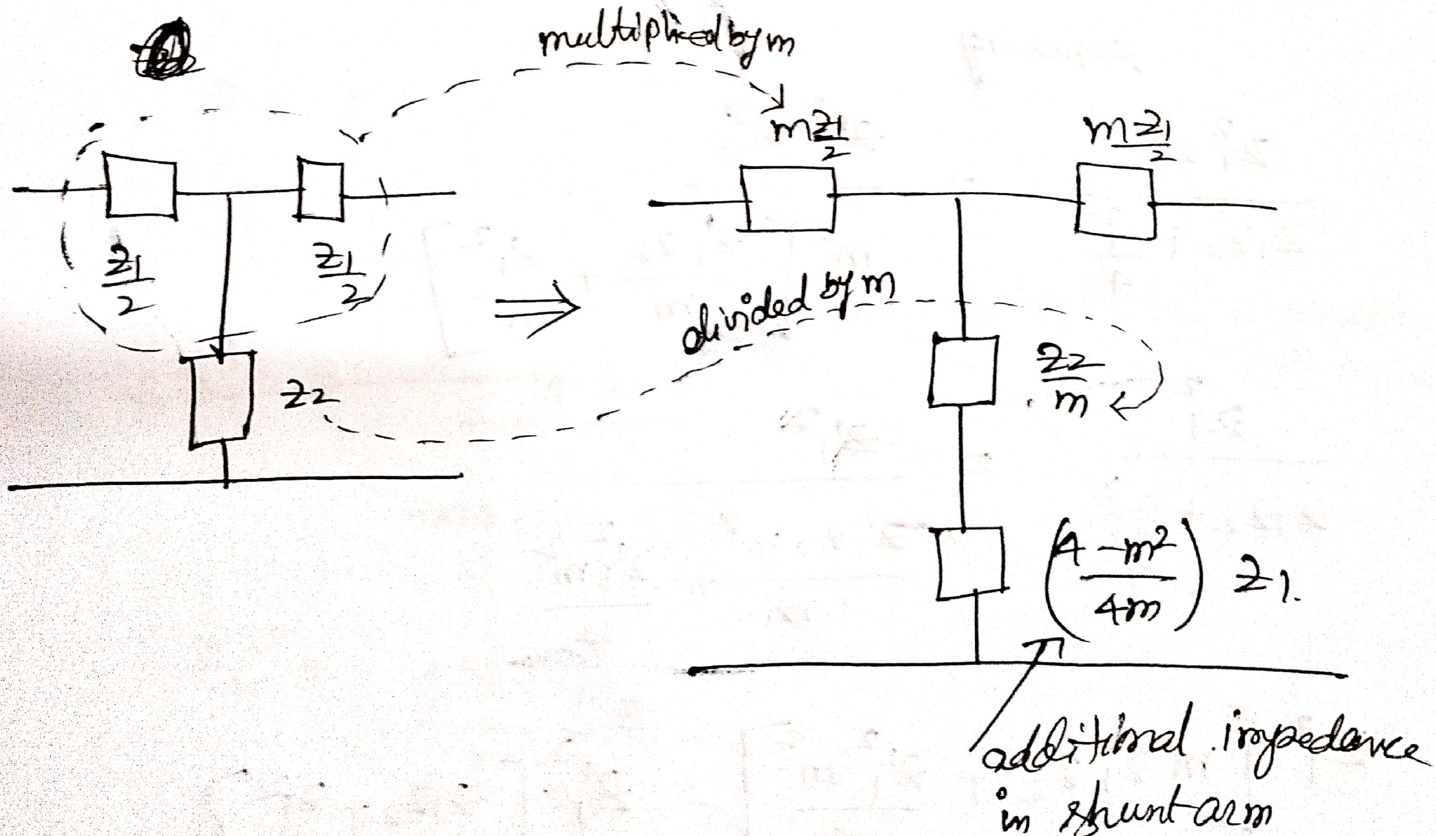
$$m Z_1 Z_2' = Z_1 Z_2 + \frac{Z_1^2}{4} - \frac{m^2 Z_1^2}{4}$$

$$m \cancel{Z_1} Z_2' = \cancel{Z_1}\left[Z_2 + \frac{Z_1}{4} - \frac{m^2 Z_1}{4}\right]$$

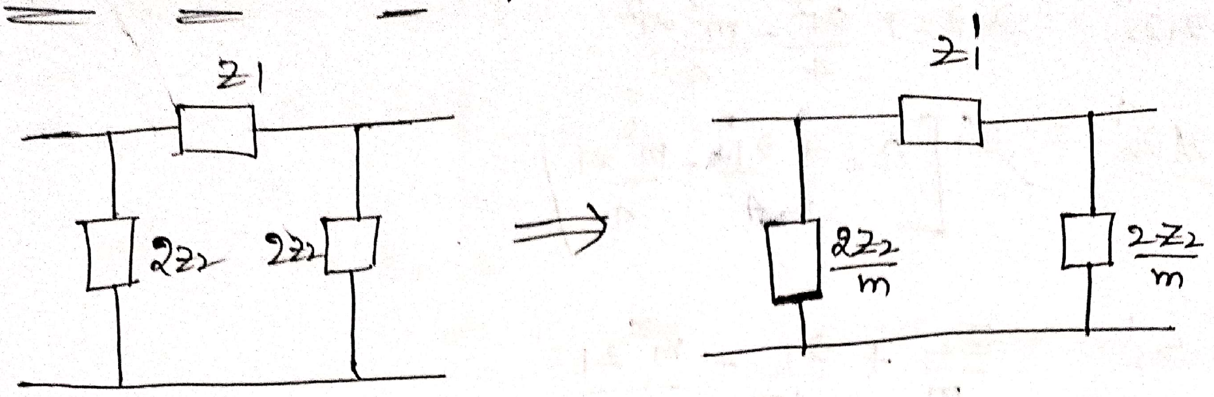
$$Z_2' = \frac{Z_2}{m} + \frac{Z_1}{4m} - \frac{m^{\cancel{2}} Z_1}{4\cancel{m}}$$

$$= Z_1\left[\frac{1}{4m} - \frac{m}{4}\right] + \frac{Z_2}{m}$$

$$Z_2' = Z_1\left[\frac{4 - m^2}{4m}\right] + \frac{Z_2}{m}$$



## m-derived π-section :




$$Z_{0\pi} = Z_{0\pi}'$$

$$Z_1 = Z_1'$$
$$Z_2 = \frac{Z_2}{m}$$

$$\frac{Z_1 Z_2}{\sqrt{Z_1 Z_2 + \frac{Z_1^2}{4}}} = \frac{Z_1' Z_2 / m}{\sqrt{\frac{Z_1' Z_2}{m} + \frac{(Z_1')^2}{4}}}$$

squaring

$$\frac{Z_1^2 \cancel{Z_2^2}}{Z_1 Z_2 + \frac{Z_1^2}{4}} = \frac{Z_1'^2 \cancel{Z_2^2}}{m^2 \left[ \frac{Z_1' Z_2}{m} + \frac{Z_1'^2}{4} \right]}$$

$$\frac{Z_1^2}{Z_1 Z_2 + \frac{Z_1^2}{4}} = \frac{Z_1'^2}{\frac{Z_1' Z_2 m^2}{\cancel{m}} + \frac{Z_1'^2 m^2}{4}}$$

$$Z_1^2 \left[ m Z_1' Z_2 + \frac{Z_1'^2 m^2}{4} \right] = Z_1'^2 \cdot \left[ Z_1 Z_2 + \frac{Z_1^2}{4} \right]$$

$$m Z_1^2 Z_1' Z_2 + \frac{Z_1^2 Z_1'^2 m^2}{4} = Z_1 Z_2 Z_1'^2 + \frac{Z_1'^2 Z_1^2}{4}$$

$$m z_1^2 z_1' z_2 + \frac{z_1^2 z_1'^2 m^2}{4} - z_1 z_2 z_1'^2 - \frac{z_1'^2 z_1^2}{4} = 0$$

$$z_1' \left[ m z_1^2 z_2 + \frac{z_1^2 z_1' m^2}{4} - z_1 z_2 z_1' - \frac{z_1' z_1^2}{4} \right] = 0$$

$$m z_1^2 z_2 + \frac{z_1^2 z_1' m^2}{4} - z_1 z_2 z_1' - \frac{z_1' z_1^2}{4} = 0$$

$$z_1' \cancel{z_1} \left[ \frac{m^2 z_1}{4} - z_2 - \frac{z_1}{4} \right] = -m z_1^2 z_2$$

$$z_1' = \frac{-m z_1 z_2}{\frac{m^2 z_1}{4} - z_2 - \frac{z_1}{4}}$$

multiply & divide denominator with $m z_1 z_2$

$$\cancel{z_1' = \frac{-m z_1 z_2}{-m z_1 z_2 \left[ \frac{m^2 z_1}{4 m z_1 z_2} \right.}}$$

$$z_1' = \frac{+m z_1 z_2}{+\left[ \frac{z_1}{4} + z_2 - \frac{m^2 z_1}{4} \right]}$$

$$z_1' = \frac{m z_1 z_2}{m z_1 z_2 \left[ \frac{z_1}{4 m z_1 z_2} + \frac{z_2}{4 m z_1 z_2} - \frac{m^2 z_1}{4 m z_1 z_2} \right]}$$

$$z_1' = \frac{1}{\underbrace{\frac{1}{z_2 m}\left(\frac{1}{4}\right)} + \frac{1}{z_1 m} - \underbrace{\frac{m}{4 z_2}}}$$

$$\cancel{z_1' = \frac{1}{\frac{1}{z_1 m} + \frac{1}{4 z_2}\left[\frac{1}{4m} - 1\right]}}$$

$$\cancel{= \frac{1}{\frac{1}{z_1 m} + \frac{1}{4 z_2}\left[\frac{1-4m}{4m}\right]}}$$

$$z_1' = \frac{1}{\frac{1}{z_1 m} + \frac{1}{4 z_2 m} - \frac{m}{4 z_2}}$$

$$z_1' = \frac{1}{\frac{1}{z_1 m} + \frac{1-m^2}{4 z_2 m}}$$

$$z_1' = \frac{1}{\frac{1}{z_1 m} + \frac{1}{z_2}\left[\frac{1-m^2}{4m}\right]}$$

$$z_1' = \frac{1}{\frac{1}{z_1 m} + \frac{1}{z_2\left[\frac{4m}{1-m^2}\right]}}$$

$$\frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \Rightarrow R_1 \parallel R_2$$

$z_1'$ is parallel of

$$(z_1 m) \parallel z_2\left(\frac{4m}{1-m^2}\right)$$

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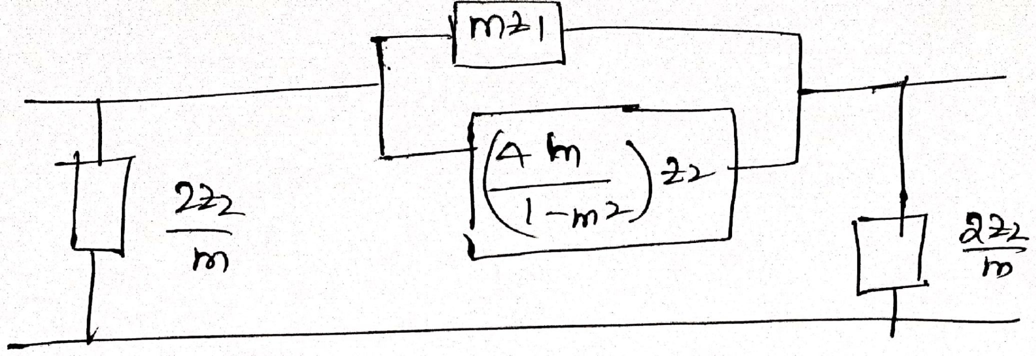
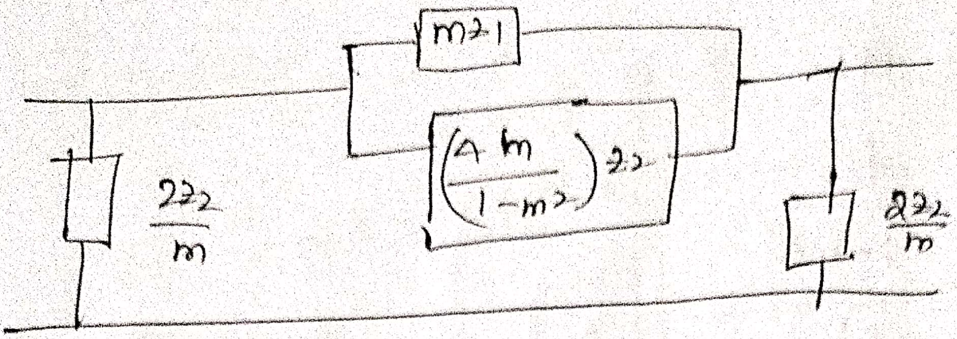
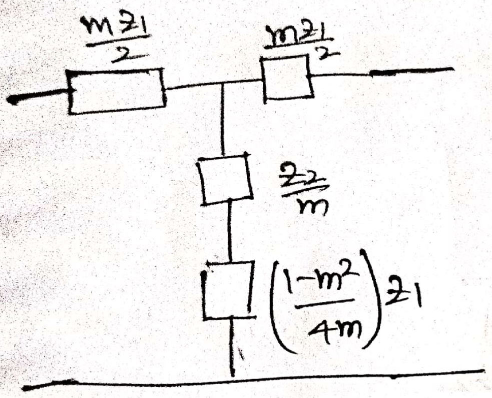


$mZ_1$
$\left(\frac{4m}{1-m^2}\right)Z_2$
$\frac{2Z_2}{m}$
$\frac{2Z_2}{m}$

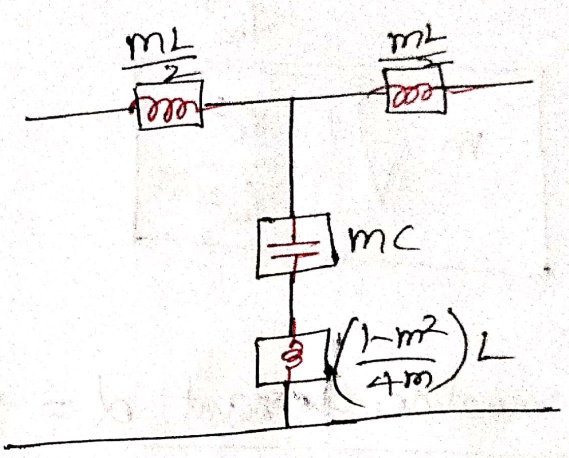



**m- derived Low pass filter:-** In LPF $Z_1 = j\omega L$ & $Z_2 = \frac{1}{j\omega C}$

m-derived T-section is given by




$\Rightarrow$




**frequency of infinite attenuation** $(f_\infty)$ is given by.

$$f_\infty = \frac{1}{2\pi\sqrt{\left(\frac{1-m^2}{4m}\right)L \cdot mC}}$$

$$= \frac{1}{2\pi\sqrt{LC \cdot \frac{\cancel{m}(1-m^2)}{4\cancel{m}}}}$$

$$= \frac{1}{2\pi\sqrt{LC}\sqrt{\frac{1-m^2}{4}}}$$

WKT for ~~T-section~~ LPF constant-k filter

$$f_c = \frac{1}{2\pi\sqrt{LC}}$$

$$f_\infty = \frac{f_c}{2\sqrt{\frac{1-m^2}{4}}}$$

$$f_\infty = \frac{f_c}{\cancel{2}/\cancel{2}\sqrt{1-m^2}}$$

$$\boxed{f_\infty = \frac{f_c}{\sqrt{1-m^2}}}$$

'm'

squaring on BS

$$f_\infty^2 = \frac{f_c^2}{1-m^2}$$

$$1-m^2 = \frac{f_c^2}{f_\infty^2}$$

$$m^2 = 1 - \frac{f_c^2}{f_\infty^2}$$

$$\boxed{m = \sqrt{1 - \frac{f_c^2}{f_\infty^2}}}$$

WKT

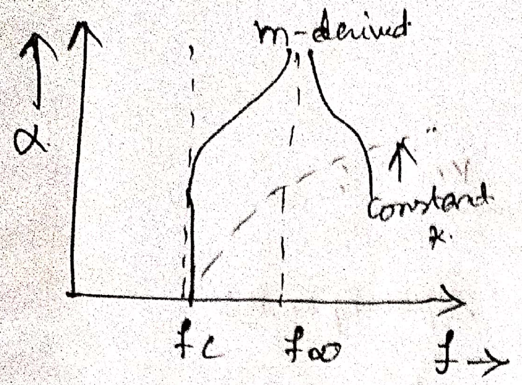
Attenuation Constant $\alpha = 2\cosh^{-1}\sqrt{\frac{Z_1}{4Z_2}}$

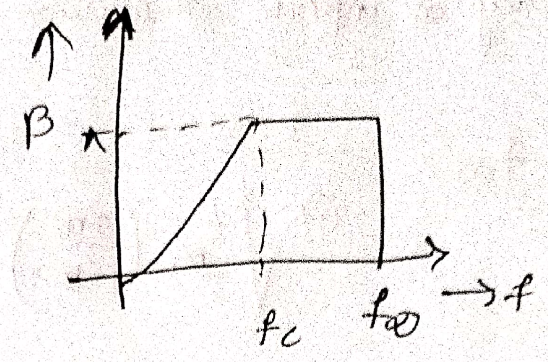
for m-derived LPF $\boxed{\alpha = 2\cosh^{-1}\frac{m\left(\frac{f}{f_c}\right)}{\sqrt{1-\left(\frac{f}{f_\infty}\right)^2}}}$

WKT

Phase Constant $\beta = 2\sin^{-1}\sqrt{\left|\frac{Z_1}{4Z_2}\right|}$

for m-derived LPF $\boxed{\beta = 2\sin^{-1}\frac{m\frac{f}{f_c}}{\sqrt{1-\left(\frac{f}{f_\infty}\right)^2}}}$



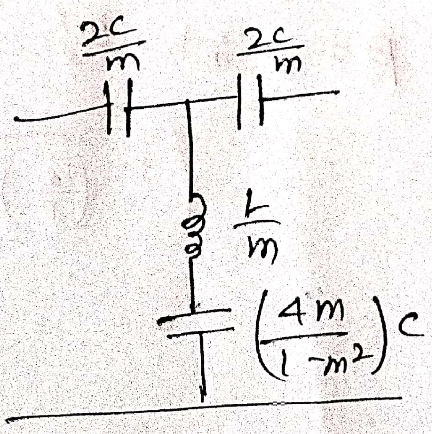



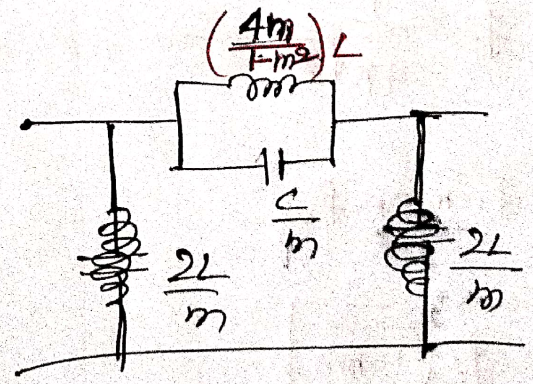

→ **Characteristic impedance for m-derived LPF**

→ for T-section $$Z_{0T} = K\sqrt{\frac{1+\frac{m^2 f^2}{c^2}}{1-\left(\frac{f}{f_\infty}\right)^2}}$$

→ for Π-section $$Z_{0\pi} = \frac{K}{\sqrt{\frac{1+\frac{m^2 f^2}{f_c^2}}{1-(f/f_\infty)^2}}}$$

**m-derived high pass filter:** $Z_1 = \frac{1}{j\omega C}$ $Z_2 = j\omega L$.



frequency at infinite attenuation

$$f_\infty = \frac{1}{2\pi\sqrt{\frac{L}{m}\left(\frac{4m}{1-m^2}\right)C}}$$

$$= \frac{1}{2\pi\sqrt{LC\cdot\left(\frac{4}{1-m^2}\right)}}$$

$$= \frac{1}{2\pi\sqrt{LC}\sqrt{\frac{(2)^2}{1-m^2}}}$$

$$= \frac{1}{4\pi\sqrt{LC}\cdot\sqrt{\frac{1}{1-m^2}}}$$

$$f_\infty = \frac{\sqrt{1-m^2}}{4\pi\sqrt{LC}}$$

for Constant K - HPF

$$f_c = \frac{1}{4\pi\sqrt{LC}}$$

$$f_\infty = \sqrt{1-m^2}\cdot f_c$$

$$\boxed{f_\infty = f_c\sqrt{1-m^2}}$$

'm' :

squaring on BS :

$$f_\infty^2 = f_c^2(1-m^2)$$

$$1-m^2 = \frac{f_\infty^2}{f_c^2}$$

$$1-\frac{f_\infty^2}{f_c^2} = m^2$$

$$\boxed{m = \sqrt{1-\frac{f_\infty^2}{f_c^2}}}$$

attenuation constant

$$\alpha = 2\cosh^{-1}\frac{m\frac{f_c}{f}}{\sqrt{1-\left(\frac{f_\infty}{f}\right)^2}}$$

Phase constant

$$\beta = 2\sin^{-1}\frac{\frac{mf_c}{f}}{\sqrt{1-\left(\frac{f_\infty}{f}\right)^2}}$$

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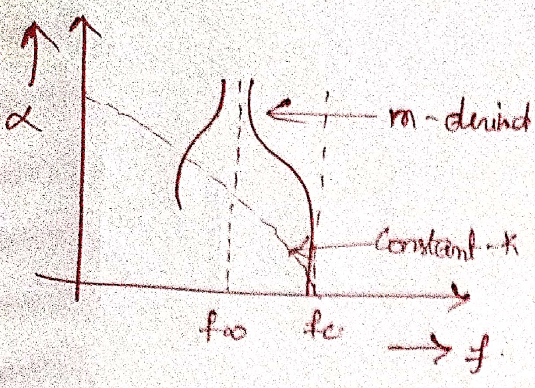
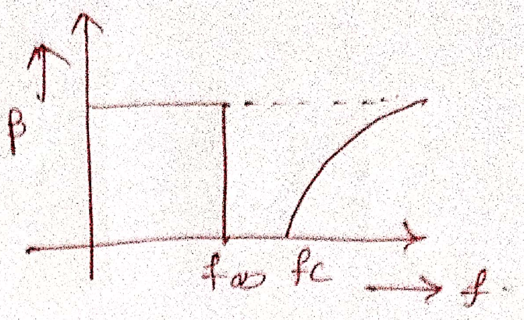



## Characteristic impedance

$$Z_{0T} = K\sqrt{1+\frac{m^2\left(\frac{f_c}{f}\right)^2}{\sqrt{1-\left(\frac{f_\infty}{f}\right)^2}}}$$

$$Z_{0\pi} = \frac{K}{\sqrt{1+\frac{m^2\left(\frac{f_c}{f}\right)^2}{\sqrt{1-\left(\frac{f_\infty}{f}\right)^2}}}}$$

**Attenuators:** Attenuators are used to reduce the signal level required in various applications in the field of electronics. For ex, attenuator may be used as volume controllers in radio station.

UNIT 5

# Network Synthesis

## Driving point impedance & Admittance function.

1) Driving point Impedance :-

⇒ It is defined for one port network.

⇒ It is defined as the ratio of laplace transform of voltage to laplace transform of current.

$$Z(s) = \frac{V(s)}{I(s)}.$$

2) Driving point admittance :-

⇒ It is defined for single port network.

⇒ It is defined as the ratio of laplace transform of current to laplace transform of Voltage.

$$Y(s) = \frac{I(s)}{V(s)}$$

## Transfer Impedance & Transfer Admittance.

Representation of basic RLC elements in s domain gives transfer impedance and transfer admittance.

1) Resistance :-

$V = iR$.

Applying LT

$V(s) = I(s) \cdot R$

$$\frac{V(s)}{I(s)} = R$$

$R \xleftrightarrow{LT} R$.

2) Inductance :-

$$V(t) = L\frac{di(t)}{dt}$$

Applying LT

$V(s) = LsI(s)$

$$\frac{V(s)}{I(s)} = Ls$$

$L \xleftrightarrow{LT} Ls$

3) Capacitance :-

$$V(t) = \frac{1}{C}\int i(t)\,dt$$

Applying LT

$$V(s) = \frac{1}{C} \cdot \frac{1}{s} I(s)$$

$$\frac{V(s)}{I(s)} = \frac{1}{Cs}$$

$C \xleftrightarrow{LT} 1/Cs$

$R \rightarrow$ Transfer impedance of resistance

$LS \rightarrow$ Transfer impedance of Inductance.

$\frac{1}{CS} \rightarrow$ Transfer impedance of capacitance

$\frac{1}{R} \rightarrow$ Transfer Admittance of resistance

$\frac{1}{LS} \rightarrow$ Transfer Admittance of Inductance

$\frac{1}{1/CS} = CS \rightarrow$ Transfer Admittance of capacitance.

Transfer function :-

It is the ratio of laplace transform of output to laplace transform input.

$$\text{Transfer function} = \frac{\text{laplace transform of output}}{\text{laplace transform of input}}.$$

$$H(s) = \frac{V_o(s)}{V_{in}(s)} \qquad H(s) = \frac{V_o(s)}{I_{in}(s)}$$

$$H(s) = \frac{I_o(s)}{I_{in}(s)} \qquad H(s) = \frac{I_o(s)}{V_{in}(s)}$$

Note :- Network function is the ratio of output to input or vice versa.

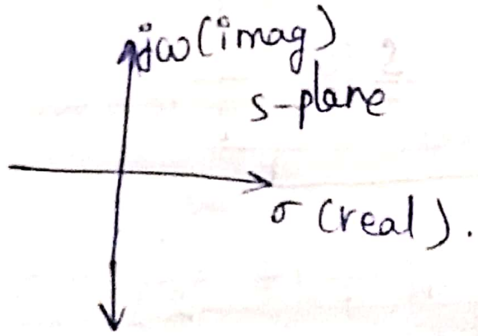
Network Analysis of poles & zeros.

⇒ By equating numerator of transfer function to zero. Zeros are determined. They are represented with 'o'.

⇒ By equating denominator of a transfer function to zero. poles are obtained. They are represented with 'x'.

⇒ poles and zeros are graphically represented in s-plane.

⇒ s-plane is the combination of real axis and imaginary axis.

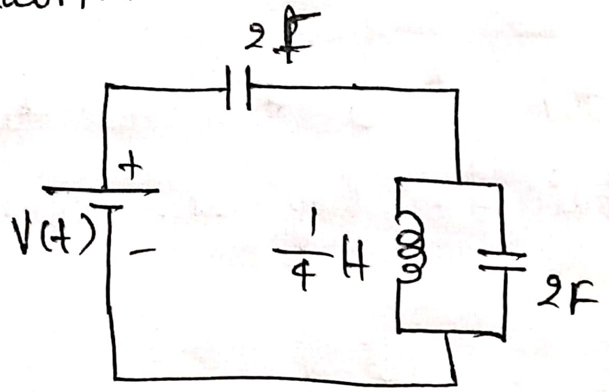



* Note: (i) stability of a system is network is defined base on the position of poles in s-plane.

(ii) If all the poles of a transfer function are on left side of a s-plane then the network is said to be stable.

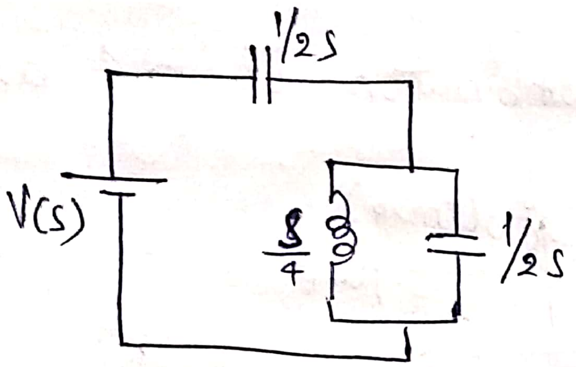
problems :-

1) find the driving point impedance $Z(s)$ of the network.




sol: convert given network into s domain. ⇒

| t domain | s-domain |
|---|---|
| R | R |
| L | Ls |
| C | $1/Cs$ |




$$Z(s) = \left(\frac{1}{2s} \parallel \frac{s}{4}\right) + \frac{1}{2s}$$

$$= \left[\frac{\frac{1}{2\cancel{s}} \cdot \frac{\cancel{s}}{4}}{\frac{1}{2s} + \frac{s}{4}}\right] + \frac{1}{2s}$$

$$= \frac{1}{8}\left[\frac{1}{\frac{4+2s^2}{8s}}\right] + \frac{1}{2s}$$

$$= \frac{1}{\cancel{8}}\left[\frac{\cancel{8}s}{4+2s^2}\right] + \frac{1}{2s}$$

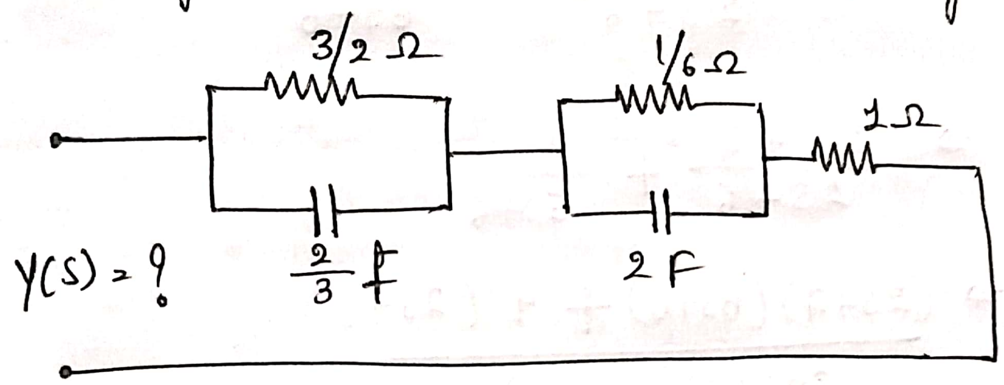
$$= \frac{s}{4+2s^2} + \frac{1}{2s} \Rightarrow \frac{2s^2+4+2s^2}{2s(4+2s^2)}$$

$$\frac{s^2+1}{\quad}$$

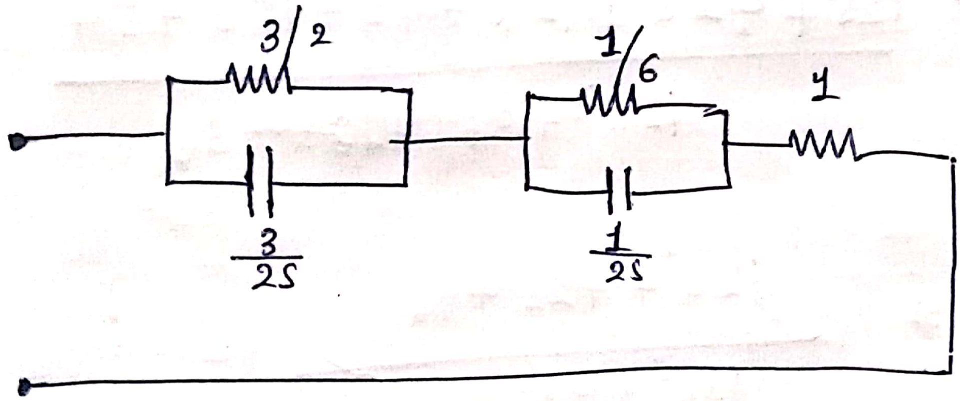
$$\Rightarrow \frac{4s^2+4}{2s(4+2s^2)} \Rightarrow \frac{4(s^2+1)}{2s \cdot 2(s^2+2)}$$

$$\therefore \boxed{Z(s) = \frac{s^2+1}{s(s^2+2)}}$$

2) find driving point admittance of given network.




Sol:- Convert n/w into s- domain.




WkT $Y(s) = \frac{1}{Z(s)}$ — ①

where

$$Z(s) = \left[1 + \left[\frac{1}{6} \middle\| \frac{1}{2s}\right]\right] + \left[\frac{3}{2} \middle\| \frac{3}{2s}\right]$$

$$Z(s) = \left[1 + \left[\frac{\frac{1}{6}\cdot\frac{1}{2s}}{\frac{1}{6}+\frac{1}{2s}}\right]\right] + \left[\frac{\frac{3}{2}\cdot\frac{3}{2s}}{\frac{3}{2}+\frac{3}{2s}}\right]$$

$$= \left[1 + \left[\frac{\frac{1}{\cancel{12s}}}{\frac{2s+6}{\cancel{12s}}}\right]\right] + \left[\frac{\frac{9}{\cancel{4s}}}{\frac{6s+6}{\cancel{4s}}}\right]$$

$$= \left[1 + \frac{1}{2s+6}\right] + \frac{9}{6s+6}$$

$$\Rightarrow \frac{2s+6+1}{2s+6} + \frac{9}{6s+6}$$

$$\Rightarrow \frac{2s+7}{2s+6} + \frac{9}{6s+6}$$

$$\Rightarrow \frac{(2s+7)(6s+6) + 9(2s+6)}{(2s+6)(6s+6)}.$$

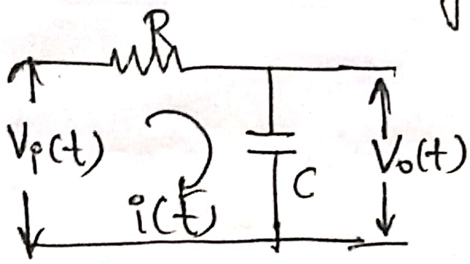
$$\Rightarrow \frac{12s^2 + 42s + 12s + 42 + 18s + 54}{12s^2 + 36s + 12s + 36}$$

$$\Rightarrow \frac{12s^2 + 72s + 96}{12s^2 + 48s + 36}$$

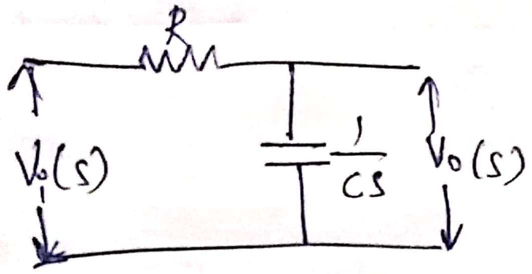
$$= \frac{\cancel{12}(s+2)(s+4)}{\cancel{12}(s+1)(s+3)}$$

$$\therefore \boxed{Z(s) = \frac{(s+2)(s+4)}{(s+1)(s+3)}}$$

3) find transfer function of given network.




Sol:- Transfer function is obtained for network in s-domain




By Transfer function = $\frac{o/p}{i/p} = \frac{V_o(s)}{V_i(s)} = H(s)$

By Applying voltage divider rule.

$$V_o(s) = \frac{V_i(s) + \frac{1}{Cs}}{R + \frac{1}{Cs}}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{Cs}\left[\frac{1}{R + \frac{1}{Cs}}\right]$$

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{Cs}\left[\frac{1}{\frac{RCs+1}{Cs}}\right]$$

$$H(s) = \frac{1}{Cs}\left[\frac{Cs}{1+RCs}\right] \Rightarrow \boxed{H(s) = \frac{1}{1+RCs}}$$

18/1/2024

Draw pole zero diagram of impedance transfer function.

$$Z(s) = \frac{s(s^2+3)(s^2+7)}{(s^2+1)(s^2+5)}.$$

Sol:- Equate Numerator to zero.

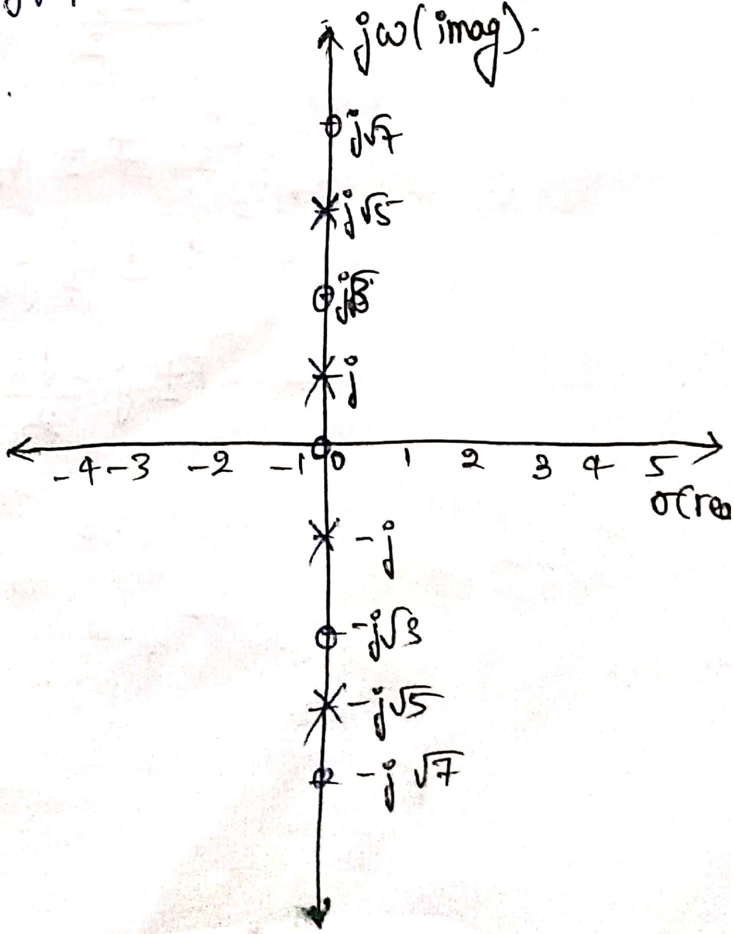
zeros:- $s(s^2+3)(s^2+7) = 0$

| | | |
|---|---|---|
| $s = 0$ | $s^2+3 = 0$ | $s^2+7 = 0$ |
| | $s^2 = -3$ | $s^2 = -7$ |
| | $s^2 = j^2 3$ | $s^2 = j^2 7$ |
| | $s = \pm j\sqrt{3}$ | $s = \pm j\sqrt{7}$ |
| | $s = +j\sqrt{3}$ | $s = +j\sqrt{7}$ |
| | $s = -j\sqrt{3}$ | $s = -j\sqrt{7}.$ |

Poles :- Equate denominator to zero.

$(s^2+1)(s^2+5) = 0$

| | |
|---|---|
| $s^2+1 = 0$ | $s^2+5 = 0$ |
| $s^2 = -1$ | $s^2 = -5$ |
| $s^2 = j^2 1$ | $s^2 = j^2 5$ |
| $s^2 = \pm j$ | $s = \pm j\sqrt{5}$ |
| $s = +j$ | $s = +j\sqrt{5}$ |
| $s = -j$ | $s = -j\sqrt{5}$ |



2) Draw pole, zero diagram of imag. impedance transfer function.

$$Z(s) = \frac{5s+4}{(s-1)(s^2+2s+4)}$$

sol:-

zeros :- Equate numerator to '0'.

$$5s + 4 = 0$$

$$s = -4/5 = -0.8.$$

poles :- Equate denominator to '0'.

$$(s-1)(s^2+2s+4) = 0.$$

$s - 1 = 0$

$s = 1$

$$s^2 + 2s + 4 = 0$$

$$s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$s = \frac{-2 \pm \sqrt{4-16}}{2}$$

$$s = \frac{-2 \pm \sqrt{-12}}{2}$$

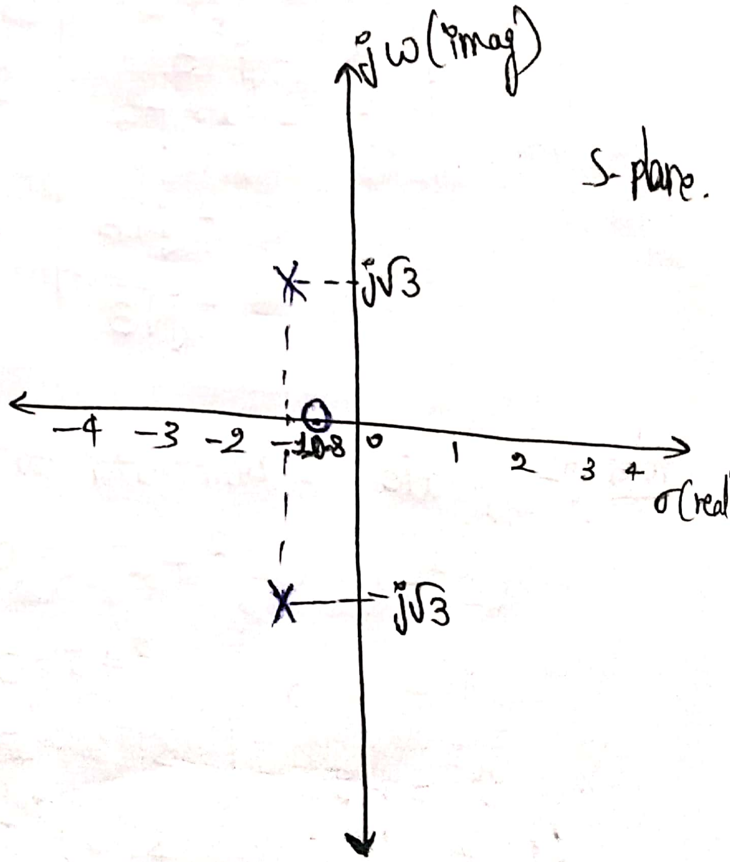
$$s = \frac{-2}{2} \pm \frac{1}{2}\sqrt{j^2 12}$$

$$= -1 \pm \frac{1}{2} j\sqrt{12}$$

$$s = -1 \pm j\frac{2\sqrt{3}}{2}$$

$$s = -1 + j\sqrt{3}$$

$$s = -1 - j\sqrt{3}.$$



* Hurwitz's polynomial (or) Hurwitz's conditions.

Hurwitz's condition (or) polynomial is one of the mathematical analysis used to synthesize a network.

⇒ Network synthesis using Hurwitz's condition is possible only when the given transfer function (or) given polynomial satisfy's hurwitz's condition.

⇒ Transfer function can be impedance transfer function (or) admittance transfer function.

⇒ Impedance (or) admittance transfer function can be the ratio of two polynomials

$$T(s) = Z(s) = \frac{P(s)}{Q(s)}.$$

⇒ If the polynomial function $P(s) = a_n s^n + a_{n-1} s^{n-1} + \dots + a_0$ is to be Hurwitz polynomial, the following conditions are to be satisfied.

short ans

Condition-1 :- All the co-efficients of given the polynomial must be positive.

Condition-2(a):- There should not be any missing term of 's' in between highest order of 's' to lowest order of 's'.

Condition-2(b):- If any 's' terms are missing then they should all be either even powers of 's' (or) odd powers of 's'.

Condition-3 :- Continued fraction expansion (CFE) of ratio of even to odd (or) odd to even must generate positive coefficients only.

Problem.

1) check whether the given polynomial is Hurwitz (or) not.

$$P(s) = s^4 + s^3 + 2s^2 + 3s + 2$$

Sol:- 1) $1, 1, 2, 3, 2 \longrightarrow +ve$

2) $s^4\ s^3\ s^2\ s^1\ s^0 \longrightarrow$ all powers are there

3) CFS.

$\frac{\text{even}}{\text{odd}}$ (or) $\frac{\text{odd}}{\text{even}}$.

$$P(s) = s^4 + s^3 + 2s^2 + 3s + 2$$

$$P_e(s) = s^4 + 2s^2 + 2$$

$$P_o(s) = s^3 + 3s.$$

$$\frac{P_e(s)}{P_o(s)} = \frac{s^4 + 2s^2 + 2}{s^3 + 3s}.$$

Apply CFE to $\frac{P_e(s)}{P_o(s)}$.

$$s^3 + 3s)\ s^4 + 2s^2 + 2\ (s \rightarrow (+ve)$$
$$\underline{s^4 + 3s^2}$$
$$-s^2 + 2)\ s^3 + 3s\ (-s \rightarrow (-ve)$$
$$\underline{s^3 - 2s}$$
$$5s)\ -s^2 + 2\ (-\tfrac{s}{5}$$
$$\underline{-s^2}$$
$$2)\ 5s\ (\tfrac{5s}{2}\ (+ve)$$
$$\underline{5s}$$
$$0$$

$\Rightarrow$ As the quotients are of +ve & -ve.

Given polynomial is not Hurwitz polynomial.

## positive real function (PRF):-

Transfer impedance and transfer admittance can be the ratio of two polynomial P(s) and Q(s) i.e., Transfer function $T(s) = Z(s) = \frac{P(s)}{Q(s)}$.

⇒ If a Transfer function is PRF then that function is physically realisable with respect to networks.

### Conditions for PRF:-

condition 1) Given transfer function must be Hurwitz transfer function i.e, both polynomials P(s) & Q(s) must be Hurwitz polynomials.

ve) a) All the co-efficients of given the polynomial must be +ve.

s b) There should not be any missing terms of 's' in between highest order of 's' to lowest order of 's'

(or)

If any 's' terms are missing then they should all be either even powers of 's' (or) odd powers of 's'

c) Continued fraction expansion (CFS) of ratio of even to odd (or) odd to even must generate positive coefficients only.

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Condition - 2 :- Real part of the transfer function must be greater than or equal $\geq 0$, for $s = j\omega$ i.e.,

$$A(\omega^2) = \left[ M_1(s) M_2(s) - N_1(s) N_2(s) \right] \geq 0 \Big|_{s = j\omega}.$$

$M_1(s)$ — even function of $P(s)$ (or) numerator.

$M_2(s)$ — even function of $Q(s)$ (or) denominator.

$N_1(s)$ — odd function of $P(s)$ (or) numerator.

$N_2(s)$ — odd function of $Q(s)$ (or) denominator.

1) Test whether $Y(s) = \dfrac{2s^3 + 2s^2 + 3s + 2}{s^2 + 1}$ is a Positive real function (or) not.

Cond-1 :- Hurwitz's condition.

Sol. $P(s) = 2s^3 + 2s^2 + 3s + 2$.

1st cond :- 2, 2, 3, 2 ——— +ve.

2nd cond :- $s^3 \; s^2 \; s^1 \; s^0$ ——— satisfy's

3rd cond :- CFE.

$P(s) = 2s^3 + 2s^2 + 3s + 2$

$P_o(s) = 2s^3 + 3s$

$P_e(s) = 2s^2 + 2$.

$$\frac{P_o(s)}{P_e(s)} = \frac{2s^3 + 3s}{2s^2 + 2} \text{ (CFE)}$$

$$
\begin{array}{l}
2s^2+2)\; 2s^3 + 3s \;( s \rightarrow +ve \\
\quad\quad\quad \cancel{2s^3} + 2s \\
\hline
\quad\quad\quad s)\; \cancel{2s^2} + 2 \;( 2s \rightarrow +ve \\
\quad\quad\quad\quad \cancel{2s^2} \\
\hline
\quad\quad\quad\quad 2)\; s \;( \frac{s}{2} \rightarrow +ve \\
\quad\quad\quad\quad\quad s \\
\hline
\quad\quad\quad\quad\quad 0
\end{array}
$$

$\therefore$ quotients are +ve

$P(s)$ is Hurwitz's polynomial

(2b). $Q(s) = s^2+1$

$1^{st}$ cond :- 1, 1 $\rightarrow$ +ve

$\boxed{2^{nd} \text{ cond :- } s^2 \quad s^0} \rightarrow$ s term missing. $\rightarrow$ $Y(s)$ is not PRF.

* $Q(s)$ can be Conditional Hurwitz's polynomial by neglecting $2^{nd}$ Condition.

$Q(s)$ is not Hurwitz's polynomial.

$\therefore Y(s)$ is not HF.

(2) $3^{rd}$ cond :- CFE

$$Q(s) = s^2+1$$

$$Q_e(s) = s^2+1$$

$$Q_o(s) = 0.$$

*

$$Q_o(s) = \frac{d}{ds} Q_e(s).$$

$$Q_o(s) = \frac{d}{ds}(s^2+1).$$

$$Q_o(s) = 2s.$$

$$\frac{Q_e(s)}{Q_o(s)} = \frac{s^2+1}{2s}.$$

$$2s)\, s^2+1 \,(\, \frac{s}{2} \rightarrow +ve.$$
$$-s^2$$
$$1)\, 2s \,(\, 2s \rightarrow +ve.$$
$$2s$$
$$0$$

$\therefore Q(s)$ is Hurwitz polynomial

$Y(s)$ is also Hurwitz transfer fn.

Condition-2 :-

$$A(\omega^2) = [M_1(s)M_2(s) - N_1(s)N_2(s)]_{s=j\omega}$$

$$M_1(s) = P_e(s) = 2s^2+2$$

$$M_2(s) = Q_e(s) = s^2+1$$

$$N_1(s) = P_o(s) = 2s^3+3s$$

$$N_2(s) = Q_o(s) = 0.$$

$$= \left[(2s^2+2)(s^2+1) - (2s^3+3s)(0)\right]_{s=j\omega}$$

$$= 2s^4 + 2s^2 + 2s^2 + 2 - 0 \Big|_{s=j\omega}$$

$$= 2s^4 + 4s^2 + 2 \Big|_{s=j\omega}.$$

$$= 2(j\omega)^4 + 4(j\omega)^2 + 2$$

$$= 2(j\omega)^4 + 4(j\omega)^2 + 2$$

$$= 2j^4\omega^4 + 4j^2\omega^2 + 2$$

$$A(\omega)^2 = 2\omega^4 - 4\omega^2 + 2$$

★ To prove

$2\omega^4 - 4\omega^2 + 2 \geq 0.$

let $\omega^2 = x$

$P_0(x) = 2x^2 - 4x + 2$

$P_1(x) = \frac{d}{dx} P_0(x) = 4x - 4$

** Divide $\frac{P_0(x)}{P_1(x)} = \frac{2x^2 - 4x + 2}{4x - 4}$

$$4x-4 \overline{)\, 2x^2 - 4x + 2 \,}\left(\frac{x}{2} - \frac{1}{2}\right.$$

$$\cancel{2x^2} - \frac{4x}{2}$$

$$- \quad +$$

$$-2x + 2$$

$$-\cancel{2x} + \frac{4}{2}$$

$$+ \quad -$$

$$(0) \rightarrow -P_2(x)$$

$-P_2(x) = 0$

$\boxed{P_2(x) = -0}$

| limits of $x$ | $P_0(x) = 2x^2 - 4x + 2$ | $P_1(x) = 4x - 4$ | $P_2(x) = -0$ | no. of sign changes |
|---|---|---|---|---|
| 0 | $+2$ | $-4$ | No change 0 | $1 + 0 = 1$ |
| $\infty$ | $+2$ | $+\infty$ | $-0$ | $0 + 1 = 1$ |

1    No change 0    1

→ overall sign change = sign change$|_{x=0}$ − sign change of $x = \infty$

$= 1 - 1 = 0$ ✓.

As overall sign change to zero.

Hence the given function is PRF.

2) Test whether $A(\omega^2) = 3\omega^4 - 12\omega^2 + 4$ is PRF (or) not.

TO

$3\omega^4 - 12\omega^2 + 4 \geq 0$.

let $\omega^2 = x$.

$P_0(x) = 3x^2 - 12x + 4$

$P_1(x) = \frac{d}{dx} P_0(x)$

$= 6x - 12$.

Divide $\frac{P_0(x)}{P_1(x)} = \frac{3x^2 - 12x + 4}{6x - 12}$

$$6x-12 \overline{)\, 3x^2 - 12x + 4 \,}\left(\frac{x}{2} - 1\right.$$

$$\cancel{3x^2} - 12x$$

$$\underline{- \quad +}$$

$$-\cancel{6x} + 4$$

$$\underline{-\cancel{6x} + 12}$$

$$\boxed{-8} \to P_2(x)$$

$+P_2(x) = +8$

$\boxed{P_2(x) = 8}$

$\Rightarrow$ overall sign change =

sign change$|_{x=0}$ − sign change$|_{x=\infty}$

$\Rightarrow 2 - 0 = 2$

As overall sign not changes to zero. Hence the given function is not PRT.

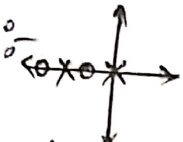
| limits of $x$ | $P_0(x) = 3x^2 - 12x + 4$ | $P_1(x) = 6x - 12$ | $P_2(x) = 8$ | no. of sign changes |
|---|---|---|---|---|
| 0 | +4   1 | −12   1 | 8 | 2 |
| $\infty$ | +4   0 | $+\infty$   0 | 8 | 0 |

Synthesis of RC, RL, LC functions.

① Identification of RC function :-

Condition-1 :- Poles & zeros must be simple (not complex). there should not be multiple poles and zeros (no number should act as both pole and zero).

condition-2 :- poles & zeros must be on -ve real axis.

condition-3 :- poles & zeros must be alternate with eachother on -ve real axis. Ex :- 

* condition-4 :- Pole should be close to origin (or) at origin.

condition-5 :- Zero should always be away from origin. i.e., $\boxed{Z(0) > Z(\infty)}$.

② Identification of RL function :-

condition-1 :- Poles & zeros must be simple (not complex). there should not be multiple poles and zeros (no number should act as both pole and zero).

Condition-2 :- poles & zeros must be on -ve real axis,

condition-3 :- poles & zeros must be alternate with eachother on -ve real axis.

* condition-4 :- pole should not be close to origin (or) not at origin.

condition-5 :- zero should always be near from origin.

$\boxed{Z(0) > Z(\infty)}$

## (8) Identification of "LC" functions.

condition-1 :- poles & zeros must be simple.

condition-2 :- poles & zeros must be on <u>imaginary axis</u>.

condition-3 :- poles & zeros must be alternate with eachother on imaginary axis.

condition-4 :- There must be either zero (or) pole at origin.

5) The highest and lowest orders of 's' of numerator & denominator must differ by 1.

1) Indicate which of the following functions belongs to RC, RL, LC impedance functions.

(a) $Z(s) = \dfrac{(s+1)(s+3)}{s(s+2)}$.

(b). $Z(s) = \dfrac{(s+1)(s+3)}{(s+2)(s+4)}$.

(c). $Z(s) = \dfrac{(s^2+1)(s^2+9)}{s(s^2+4)}$.

(a). $Z(s) = \dfrac{(s+1)(s+3)}{s(s+2)}$

poles :- equate denominator to zero.

$s(s+2) = 0$

$s = 0 \mid s+2 = 0$

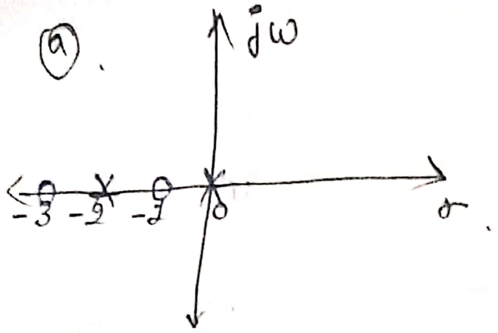
$-2$.

zeroes = equate numerator to zero.

$s+1 = 0 \quad s+3 = 0$

$s = -1 \quad s = -3$.

(a).




poles

$s(s+2)=0$

$s=0$

$s=-2$

zeros.

$(s+1)(s+3)=0$

$s=-1$

$s=-3$.

As pole is at origin & all poles & zeros are on (-ve) real axis.

Given function belongs to RC.

(b). poles:

$(s+2)(s+4)=0$

$s=-2$

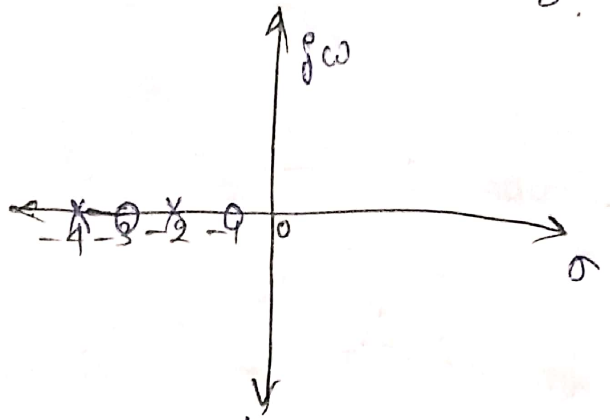
$s=-4$

zeros.

$(s+1)(s+3)=0$

$s=-1$

$s=-3$.




As ~~pole~~ zero is at near to origin & all poles & zeros are on (-ve) real axis

Given function belongs to RL

(c). poles

$s(s^2+4)=0$

$s=0$

$s^2+4=0$

$s=\sqrt{-4}$

$s=j\sqrt{4}$

$s=+j2$

$s=-j2$

zeros

$(s^2+1)(s^2+9)=0$

$s^2+1=0$

$s^2=-1$

$s=\sqrt{-1}$

$s=\pm j\sqrt{1}$

$s=+j\sqrt{1}=+j$

$s=-j\sqrt{1}=-j$

$s^2+9=0$

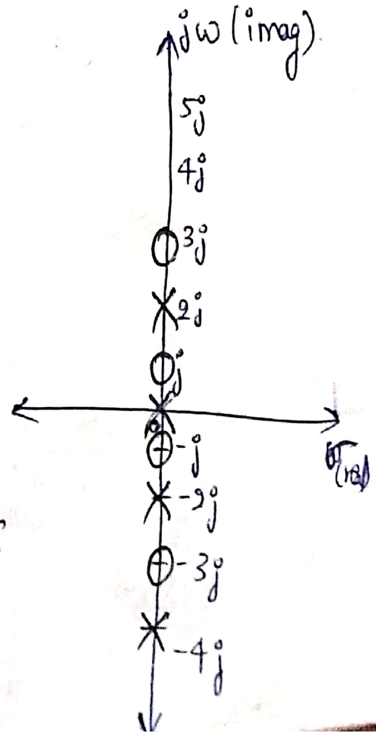
$s^2=-9$

$s=\sqrt{-9}$

$s=j\sqrt{9}$

$s=+j3$

$s=-j3$.




→ As pole at origin and all poles & zeros are on imaginary axis then the given function belongs to 'LC'.

Home work

(P1). Test if polynomial $s^3+6s^2+12s+8$ is Hurwitz.

(P2). Determine whether function $F(s) = \frac{s^2+6s+5}{s^2+9s+14}$ is PRF (or) not.

(P3). Identify which of the following belongs to RC, RL, LC impedance functions.

(a) $Z(s) = \frac{2(s+1)(s+3)}{(s+2)(s+6)}$

(b) $Z(s) = \frac{(s+1)(s+3)}{s(s+2)(s+4)}$

(c) $Z(s) = \frac{s^3+2s}{s^4+4s^2+3}$.

(P1) Ans. $P(s) = s^3+6s^2+12s+8$ → Hurwitz conditions.

(i) 1, 6, 12, 8 ⇒ all are +ve having co-efficients.

(ii) $s^3, s^2, s^1, s^0$ ⇒ no sterms missing.

(iii) continued fraction exapansion (CFE).

$\frac{\text{even}}{\text{odd}}$ (or) $\frac{\text{odd}}{\text{even}}$.

$P(s) = s^3+6s^2+12s+8$

$P_o(s) = s^3+12s$

$P_e(s) = 6s^2+8$

$$\frac{P_o(s)}{P_e(s)} = \frac{s^3+12s}{6s^2+8}.$$

$6s^2+8)\ s^3+12s\ (\frac{s}{6} \rightarrow +ve$

$\cancel{s^3}+\frac{8s}{6}$ [4/3]

$\frac{32}{3}s)\ 6s^2+8\ (\frac{18}{32}s \rightarrow +ve$

$\cancel{6s^2}$

$8)\ \frac{32}{3}s\ (\frac{4}{3}s \rightarrow +ve.$

$\frac{32}{3}s$

$(0).$

∴ since all quotients are +ve

It obeys all the conditions.

∴ Given function is Hurwitz polynomial.

(P2). $F(s) = \frac{s^2+6s+5}{s^2+9s+14}$

Cond 1 :- Hurwitz condition.

$$F(s) = \frac{P(s)}{Q(s)} = \frac{s^2+6s+5}{s^2+9s+14}$$

(1a). $P(s) = s^2+6s+5$

(i) 1, 6, 5 ⇒ all are +ve.

(ii) $s^2\ s^1\ s_0$ ⇒ No s terms missing.

(iii) CFE $P(s) = s^2+6s+5$

$P_e(s) = s^2+5$

$P_o(s) = 6s.$

$$\frac{P_e(s)}{P_o(s)} = \frac{s^2+5}{6s}$$

$47 s^2 A = \frac{\ }{r} s$ $\quad 3/12/1$

$$6s \overline{)\, s^2 + 5\,} \left(\frac{s}{6} \longrightarrow +ve\right.$$
$$\underline{-\cancel{s^2} \quad -}$$
$$5 \overline{)\, 6s\,} \left(\frac{6}{5}s \longrightarrow +ve.\right.$$
$$\underline{6s}$$
$$0$$

$\therefore$ $P(s)$ is Hurwitz's polynomial.

1b. $Q(s) = s^2 + 9s + 14.$

(i) $1, 9, 14 \Rightarrow$ all +ve.

(ii) $s^2, s_1, s_0 \Rightarrow$ no s terms missing.

(iii) CFE $\quad Q(s) = s^2 + 9s + 14$

$$Q_e(s) = s^2 + 14$$
$$Q_o(s) = 9s$$

$$\frac{Q_e(s)}{Q_o(s)} = \frac{s^2+14}{9s}$$

CFE :- $9s \overline{)\, s^2 + 14\,} \left(\frac{s}{9} \longrightarrow +ve\right.$
$$\underline{-\cancel{s^2} \quad -}$$
$$14 \overline{)\, 9s\,} \left(\frac{9}{14} s \rightarrow +ve\right.$$
$$\underline{9s}$$
$$\times$$

$\therefore$ all quotients are +ve

$\therefore Q(s)$ is Hurwitz's polynom

$F(s)$ is Hurwitz fn.

Cond-2 :-

$$A(\omega^2) = \left[ M_1(s)\, M_2(s) - N_1(s) \cdot N_2(s) \right] \Big|_{s = j\omega}.$$

$$M_1(s) = P_e(s) = s^2 + 5$$
$$M_2(s) = Q_e(s) = s^2 + 14$$
$$N_1(s) = P_o(s) = 6s$$
$$N_2(s) = Q_o(s) = 9s$$

$$A(\omega^2) = \left[ (s^2+5)(s^2+14) - 6s(9s) \right] \Big|_{s=j\omega}$$

$$= s^4 + 14s^2 + 5s^2 + 70 - 54s^2 \Big|_{s=j\omega}$$

$$= s^4 - 35s^2 + 70 \Big|_{s=j\omega}$$

$$A(\omega^2) = (j\omega)^4 - 35(j\omega)^2 + 70$$

$$A(\omega^2) = \omega^4 + 35\omega^2 + 70$$

To prove $\omega^4 + 35\omega^2 + 70 \geq 0$, let $\omega^2 = x$.

$$P_0(x) = x^2 + 35x + 70$$

$$P_1(x) = \frac{d}{dx} P_0(x)$$

$$= \frac{d}{dx}\left[x^2 + 35x + 70\right]$$

$$P_1(x) = 2x + 35$$

Now divide $\frac{P_0(x)}{P_1(x)} = \frac{x^2 + 35x + 70}{2x + 35}$

$$2x+35 \,\big)\, x^2 + 35x + 70 \,\left(\frac{x}{2} + \frac{35}{4}\right.$$

$$\underline{\cancel{x^2} + \frac{35}{2}x}$$

$$\cancel{\frac{35}{2}x} + 70$$

$$\underline{\cancel{\frac{35}{2}x} + \frac{1225}{4}}$$

$$-\frac{945}{4} \Rightarrow -P_2(x).$$

$$-P_2(x) = -\frac{945}{4} \Rightarrow \boxed{P_2(x) = \frac{945}{4}}$$

| limits of $x$ | $P_0(x) = x^2+35x+70$ | $P_1(x) = 2x+35$ | $P_2(x) = \frac{945}{4}$ | change in sign |
|---|---|---|---|---|
| 0 | $+70$ $\quad 0 = NC$ | $+35$ $\quad 0 = NC$ | $+\frac{945}{4}$ | 0 |
| $\infty$ | $+\infty$ $\quad 0 = NC$ | $+\infty$ $\quad 0 = NC$ | $+\frac{945}{4}$ | 0 |

overall change $=$ sign change$\big|_{x=0}$ $-$ sign change$\big|_{x=\infty}$

$$= 0 - 0 = 0.$$

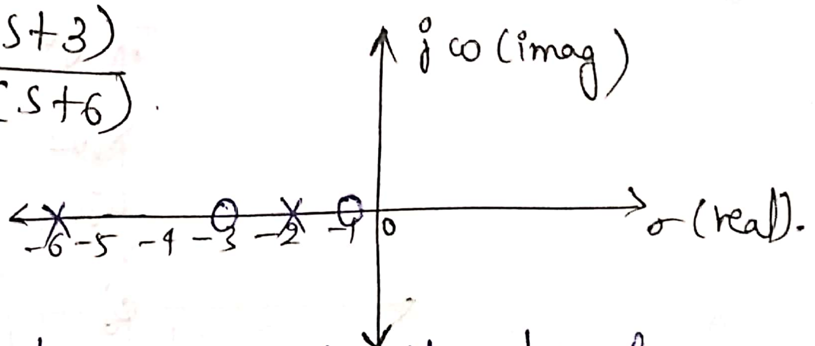
$\therefore$ overall change is zero.

Then the given $f^n$ is PRF.

(P3). a) $$Z(s) = \frac{2(s+1)(s+3)}{(s+2)(s+6)}.$$

zeros $= -1, -3$

poles $= -2, -6$.

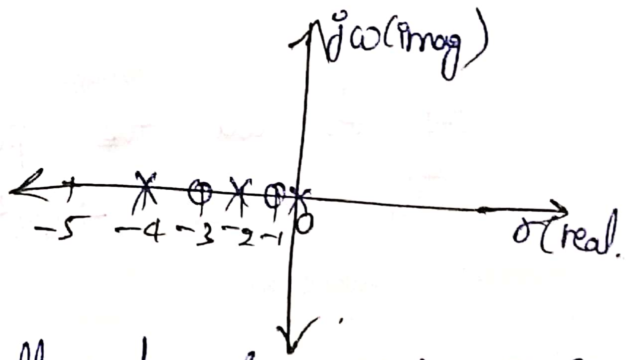



As zero is nearer to origin & all poles & zeros are on $-ve$ real axis. The given $f^n$ belongs to RL ckt.

(b). $$Z(s) = \frac{(s+1)(s+3)}{s(s+2)(s+4)}.$$

zeros $= -1, -3$

poles $= 0, -2, -4$.




As pole is at origin & all poles & zeros are on $-ve$ real axis. The given function belongs to RC ckt.

(c). $z(s) = \dfrac{s^3+2s}{s^4+4s^2+3}$

$z(s) = \dfrac{s(s^2+2)}{(s^2+3)(s^2+1)}$

poles = $(s^2+3)(s^2+1) = 0$

$s^2 = -3$

$s = \pm j\sqrt{3}$

$s = +j\sqrt{3}$

$s = -j\sqrt{3}$

$s^2 = -1$

$s = \pm j$

$s = +j$

$s = -j$

zero = $s(s^2+2) = 0$

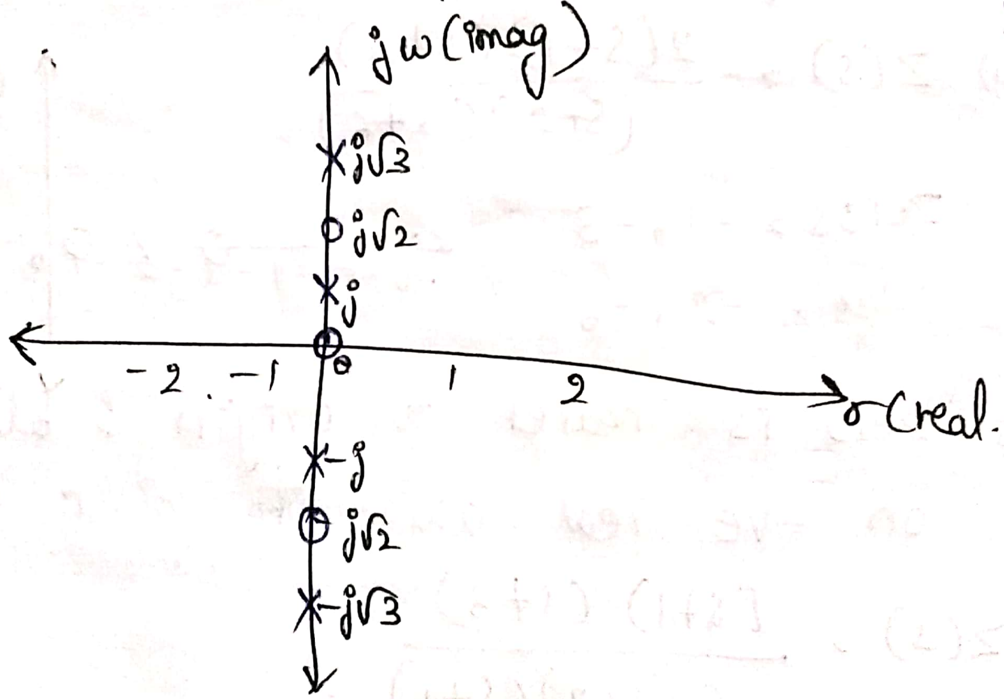
$s = 0 \quad s^2+2 = 0$

$s^2 = -2$

$s = \pm j\sqrt{2}$

$s = +j\sqrt{2}$

$s = -j\sqrt{2}$.




⇒ As all the poles & zeros lies on imaginary axis the given function belongs to '$L_C$'.

functions
Synthesis of RC, RL, LC using Cauer 19/1/2024.
method.

Cauer method is applied in two - cases where in each Cauer method is of 2 - types.

Case Cauer method

Case(i) :- If the degree of numerator and degree of denominator are not equal. Then procedural steps to be followed for Cauer - I & Cauer - II are

| Cauer - I | Cauer - II |
|---|---|
| step-1 :- choose $Z(s)$ (or) $Y(s)$ such that degree of numerator will be greater than degree of denominator $D(N) > D(D)$. | S-1 :- choose $Z(s)$ or $Y(s)$ such that degree of numerator should be less than degree of denominator. $D(N) < D(D)$. |
| step-2 :- Arrange order of numerator & denominator from highest order to lowest order. | Step 2 :- Arrange order of numerator & denominator from lowest to highest order. |
| step-3 :- Perform continued fraction expansion. | Step-3 :- same as Cauer - I |
| step-4 : ① If $Y(s)$ is considered then coeff quotients are as followed | step- 4 :- same as Cauer - I |

| $Y(s)$ | $Z(s)$ |
|---|---|
| $q_1 = Y_1$ | $q_1 = Z_1$ |
| $q_2 = Z_2$ | $q_2 = Y_2$ |
| $q_3 = Y_3$ | $q_3 = Z_3$ |
| $q_4 = Z_4$ | $q_4 = Y_4$ |
| $q_5 = Y_5$ | |

Step-5 :- Identifying impedance elements based on following table.

| Q (quotient) form | Z(s) | Y(s). |
|---|---|---|
| a | a | 1/a |
| as | a | a |
| $\frac{a}{s}$ | 1/a | 1/a |

* a → Any real constant value.

Step-6 :- Design the circuits in such away that

Case (i) :- If $Q_1 = Y_1$, then start the circuit with shunt impedance.

Case (ii) :- If $Q_1 = Z_1$, then start the circuit with series impedance.

Step-5 :- Same as Cauer-I

Step-6 :- Design the ckts in such away that.

① If $Q_1 = Y_1$ → series

② $Q_1 = Z_1$ → shunt

Case (iii) :- If degree of numerator & denominator are equal.

Then produre to follow Cauer-I & Cauer-II are :-

## Cauer - I

**step-1 :-** Identify poles & zeros. and also find whether poles are close to origin (or) zeros are close to origin.

**step-2 :-** choose $Z(s)$ & $Y(s)$ in such away that.

case(i) :- If zero is close to origin, then consider $Y(s)$

case(ii) :- If pole is close to origin, then consider $Z(s)$.

⇒ order → highest to lowest.

**step-3 :-** Perform CFE.

**step-4 :-**

| $Y(s)$ | $Z(s)$ |
|---|---|
| $Q_1 = Y_1$ | $Q_1 = Z_1$ |
| $Q_2 = Z_2$ | $Q_2 = Y_1$ |
| $Q_3 = Y_3$ | $Q_3 = Z_3$ |
| ⋮ | ⋮ |

**step-5 :-** Identify impedance elements based on following table.

| Q form | $Z(s)$ | $Y(s)$ |
|---|---|---|
| $a$ | —ww— $a$ | —ww— $1/a$ |
| $as$ | —ooo— $a$ | —\|\|— $a$ |
| $\frac{a}{s}$ | —\|\|— $1/a$ | —ooo— $1/a$ |

**step-6 :-** Design the ckts in such away that

⇒ If $Q_1 = Y_1$ ⇒ shunt

## Cauer - II

**Step-1 :-** Same as Cauer-I

**Step-2 :-** choose $Z(s)$ & $Y(s)$ in such away that.

If zero is close to origin, then consider $Z(s)$.

If pole is close to origin then consider $Y(s)$.

⇒ order lowest to highest.

**step 3 :-** Same as Case (i)

Step-4 :-

Step-5 :- → Cauer I.

Step-6 :-

$Q_1 = Y_1$ = series

$Q_1 = Z_1$ = shunt.

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## Cauer - I

**step-1 :-** Identify poles & zeros. and also find whether poles are close to origin (or) zeros are close to origin.

**step-2 :-** choose $Z(s)$ & $Y(s)$ in such away that.

case(i) :- If zero is close to origin, then consider $Y(s)$

case(ii) :- If pole is close to origin, then consider $Z(s)$.

⇒ order → highest to lowest.

**step-3 :-** Perform CFE.

**step-4 :-**

| $Y(s)$ | $Z(s)$ |
|---|---|
| $q_1 = Y_1$ | $q_1 = Z_1$ |
| $q_2 = Z_2$ | $q_2 = Y_1$ |
| $q_3 = Y_3$ | $q_3 = Z_3$ |
| ⋮ | ⋮ |

**step-5 :-** Identify impedance elements based on following table.

| q form | Z(s) | Y(s) |
|---|---|---|
| $a$ | $a$ | $1/a$ |
| $as$ | $a$ | $a$ |
| $\frac{a}{s}$ | $1/a$ | $1/a$ |

**step-6 :-** Design the ckts in such away that

⇒ If $q_1 = Y_1$ ⇒ shunt

$q_1 = Z_1$ ⇒ series.

## Cauer - II

**Step-1 :-** As same as Cauer-I

**Step-2 :-** choose $Z(s)$ & $Y(s)$ in such away that.

If zero is close to origin, then consider $Z(s)$.

If pole is close to origin then consider $Y(s)$.

⇒ order lowest to highest.

**Step 3 :-** Same as Case (i)

Step-4 :-
Step-5 :-
Step-6 :- } → Cauer I.

$q_1 = Y_1$ = series

$q_1 = Z_1$ = shunt.

Problems:-

1) obtain Cauer- I & Cauer - II synthesis for

$$Z(s) = \frac{(s+1)(s+4)}{s^3+7s^2+10s}.$$

Sol:- As order of numerator $\neq$ order of denominator.

Synthesis carry with Case (1).

Cauer- I

$S_1$ :- $D(N) > D(D)$

Given that $Z(s) = \dfrac{(s+1)(s+4)}{s^3+7s^2+10s}$.

As $D(N) \not> D(D)$.

Consider $Y(s)$.

$$Y(s) = \frac{1}{Z(s)} = \frac{s^3+7s^2+10s}{(s+1)(s+4)}$$

$S_2$ :- highest - lowest.

$$Y(s) = \frac{s^3+7s^2+10s}{s^2+5s+4}$$

| Form | $Z(s)$ | $Y($ |
|---|---|---|
| $a$ | $a$ | |
| $as$ | $a$ | |
| | $1/$ | |

CFE :-

$$s^2+5s+4 \overline{\smash{)}\, s^3+7s^2+10s} \left( s \rightarrow Q_1 \right.$$
$$\underline{s^3+5s^2+4s}$$
$$2s^2+6s \overline{\smash{)}\, s^2+5s+4} \left( \tfrac{1}{2} \rightarrow Q_2 \right.$$
$$\underline{s^2+\tfrac{6}{2}3s}$$
$$2s+4 \overline{\smash{)}\, 2s^2+6s} \left( s \rightarrow Q_3 \right.$$
$$\underline{2s^2+4s}$$
$$2s \overline{\smash{)}\, 2s+4} \left( 1 \rightarrow Q_4 \right.$$
$$\underline{2s}$$
$$4 \overline{\smash{)}\, 2s} \left( \tfrac{s}{2} \downarrow Q_5 \right.$$
$$\underline{-2s}$$
$$0$$

S4 :- As $Y(s)$ is considered.

$Q_1 = Y_1 = s$

$Q_2 = Z_2 = 1/2$

$Q_3 = Y_3 = s$

$Q_4 = Z_4 = 1$

$Q_5 = Y_5 = \frac{s}{2}$

S5 :- Identify the impedance elements.

$Y_1 = s = 1 \cdot s$ (as) $\Rightarrow$ as form w.r.t $Y$ is capacitor $a$ $\Rightarrow$ $1F$

$Z_2 = 1/2\,(a) \Rightarrow a$ form w.r.t $z$ is resistor $a$ $\Rightarrow$ $1/2\,\Omega$.

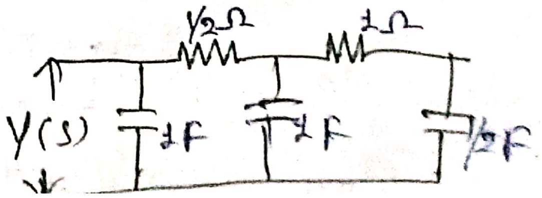
$Y_3 = s \Rightarrow 1(s)$ (as) $\Rightarrow$ as w.r.t $Y$ is capacitor $a$ $\Rightarrow$ $1F$

$Z_4 = 1$ (a) $= a$ form w.r.t to $z$ is resistor $a$ $\Rightarrow$ $1\,\Omega$

$Y_5 = \frac{s}{2} = \frac{1}{2}(s) =$ as form w.r.t to $Y$ is capacitor $a$ $\Rightarrow$ $1/2\,F$

S6 :- As $Q_1 = Y_1$

ckt starts with shunt impedance.



(9). Cauer - II Case (i).

(1). $Z(s) = \dfrac{(s+1)(s+4)}{s^3+7s^2+10s}$.

$$D(N) < D(D)$$

Consider. $Z(s) = \dfrac{(s+1)(s+4)}{s^3+7s^2+10s} = \dfrac{s^2+5s+4}{s^3+7s^2+10s}$.

(2). Arrange the order lowest to highest.

$$Z(s) = \frac{4+5s+s^2}{10s+7s^2+s^3}$$

(3). CFE :-

$10s+7s^2+s^3 \,)\, 4+5s+s^2 \,\left(\dfrac{4}{10s} \rightarrow Q_1\right.$

$\quad -\cancel{4} + \dfrac{14}{5}s + \dfrac{2s^2}{5}$

$\quad \dfrac{11}{5}s + \dfrac{3}{5}s^2 \,)\, 10s+7s^2+s^3 \,\left(\dfrac{50}{11} \rightarrow Q_2\right.$

$\quad\quad -\cancel{10s} + \dfrac{30}{11}s^2$

$\quad\quad \dfrac{47}{11}s^2+s^3 \,)\, \dfrac{11}{5}s + \dfrac{3}{5}s^2 \,\left(\dfrac{121}{235s} \rightarrow Q_3\right.$

$\quad\quad\quad -\cancel{\dfrac{11}{5}s} + \dfrac{121}{235}s^2$

$\quad\quad\quad \dfrac{4}{47}s^2 \,)\, \dfrac{47}{11}s^2 + s^3 \,\left(\dfrac{2209}{44}\right.$

$\quad\quad\quad\quad -\cancel{\dfrac{47}{11}s^2}$

$\quad\quad\quad\quad s^3 \,)\, \dfrac{4}{47}s^2 \,\left(\dfrac{4}{47}\right.$

$\quad\quad\quad\quad\quad \cancel{\dfrac{4}{47}s^2}$

$\quad\quad\quad\quad\quad (0)$

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(S4) As $Z(S)$ is considered

$$q_1 = Z_1 = \frac{4}{10S}$$

$$q_2 = Y_2 = \frac{50}{11}$$

$$q_3 = Z_3 = \frac{121}{235S}$$

$$q_4 = Y_4 = \frac{2209}{44}$$

$$q_5 = Z_5 = \frac{4}{47S}$$

(S5). Identify the impedance elements.

$Z_1 = \frac{4}{10S} \Rightarrow \frac{a}{S}$ form w.r.t $Z$ — $1/a$ $\Rightarrow$ $\frac{10}{4}F$

$Y_2 = \frac{50}{11} \Rightarrow a$ form w.r.t $Y$ — $1/a$ $\Rightarrow$ $11/50\,\Omega$

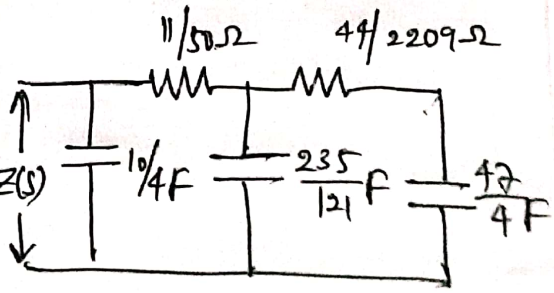
$Z_3 = \frac{121}{235S} \Rightarrow \frac{a}{S}$ form w.r.t $Z$ — $1/a$ $\Rightarrow$ $\frac{235}{121}F$

$Y_4 = \frac{2209}{44} \Rightarrow a$ w.r.t $Y$ — $1/a$ $\Rightarrow$ $\frac{44}{2209}\Omega$

$Z_5 = \frac{4}{47S} \Rightarrow \frac{a}{S}$ w.r.t $Z$ — $1/a$ $\Rightarrow$ $\frac{47}{4}F$.

$q_4$ (6). As $Q_1 = Z_1$

$q_5$ ckt start with shunt impedance



Synthesis of RL, RC & LC functions using foster method.

There exist 2 - types of foster methods. named as foster - I & foster - II.

| Foster - I | Foster - II. |
|---|---|
| Step-1:- Transfer function must be Z(s) only. | ①. Transfer function must be Y(s) only. |
| Step-2:- Denominator of Z(s) must be in the form of factors. But not the polynomials. | ②. Denominator of Y(s) must be in the form of factors. But not the polynomial |
| Step-3:- Apply Partial fraction Expansion (PFE). | ③ Apply partial fraction Expansion (PFE. |
| Step-4:- Identify the impedance elements based on the result of Partial fraction as listed below. | ④. Identify the impedance elements based on the result of partial fraction as listed below |

| P.F.E form. | Z(s) |
|---|---|
| $a$ | $a$ |
| $as$ | $a$ |
| $\frac{a}{s}$ | $1/a$ |
| $\frac{a}{s+b}$ | $\frac{a}{s}$ $\frac{a}{b}$ — $1/a$, $a/b$ |
| $\frac{as}{s+b}$ | $\frac{as}{s}$ $\frac{as}{b}$ — $a/b$ |
| $\frac{as}{s^2+b}$ | $\frac{as}{s^2}$ $\frac{as}{b}$ — $1/a$, $a/b$ |

| PFE form | Y(s). |
|---|---|
| $a$ | $1/a$ |
| $as$ | $a$ |
| $\frac{a}{s}$ | $1/a$ |
| $\frac{a}{s+b}$ | $1/a$, $a/b$ |
| $\frac{as}{s+b}$ | $a/b$, $1/a$ |
| $\frac{as}{s^2+b}$ | $1/a$, $a/b$ |

| | |
|---|---|
| 4. step-5 :- connect all the elements of step-4 in series to design the ckt. | Connect all the elements of step-4 in shunt (or) parallel to design the ckt. |

4) Synthesize the give Transfer function using foster-I & foster-II methods.

$$Z(s) = \frac{4(s^2+1)(s^2+16)}{s(s^2+4)}$$

Foster-I

sol :- $S_1$ :- $Z(s) = \frac{4(s^2+1)(s^2+16)}{s(s^2+4)}$

$S_2$ :- Denominator has factors.

$S_3$ :- PFE.

$$\frac{4(s^2+1)(s^2+16)}{s(s^2+4)} = \frac{A}{s} + \frac{Bs}{s^2+4}$$

$$\frac{4(s^2+1)(s^2+16)}{s(\cancel{s^2+4})} = \frac{As^2+4A+Bs^2}{\cancel{s(s^2+4)}}$$

$$4[s^4+17s^2+16] = s^2[A+B]+4A$$

$$4s^4+68s^2+64 = (A+B)s^2+4A$$

Compare $s^2$, constant.

$A+B = 68$

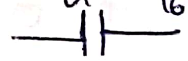
$4A = 64 \Rightarrow A = \frac{64}{4} \Rightarrow \boxed{A=16}$

$16+B = 68 \Rightarrow \boxed{B=52}$

$$Z(s) = \frac{16}{s} + \frac{52s}{s^2+4}$$

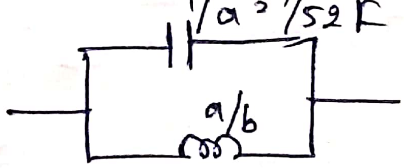
(S4) :-

$Z(s) = \frac{16}{s}$ $\frac{1}{a} = \frac{1}{16}$ F



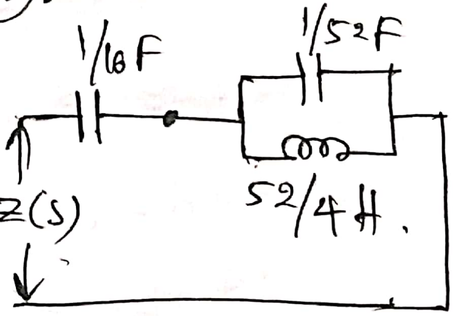
$Z(s) = \frac{52s}{s^2+4} = \frac{as}{s^2+b}$

$1/a = 1/52$ F




$52/4$ H.

(S5).




//.

## Foster – II

Given $Z(s) = \dfrac{4(s^2+1)(s^2+16)}{s(s^2+4)}$

but in foster - II, we should consider $Y(s)$ only.

(S1):- $Y(s) = \dfrac{1}{Z(s)} = \dfrac{s(s^2+4)}{4(s^2+1)(s^2+16)}$

(S2) denominator as factors.

(S3):- PFE.

$$\frac{s(s^2+4)}{4(s^2+1)(s^2+16)} = \frac{As}{s^2+1} + \frac{Bs}{s^2+16}$$

$$\frac{1}{4}\left[\frac{s(s^2+4)}{4(s^2+1)(s^2+16)}\right] = \frac{As^3+16As+Bs^3+Bs}{(s^2+1)(s^2+16)}$$

$$s^3+4s = As^3+16As+Bs^3+Bs$$

$$s^3+4s = s^3(A+B)+s(16A+B).$$

Compare $s^3$ & $s$.

$16 \times \quad A+B = 1$

$16A+B = 4$

$\cancel{16A} + 16B = 16$

$\cancel{16A} + B = 4$

$15B = 12$

$$\boxed{B = \frac{12}{15}}$$

$A + \frac{12}{15} = 1$

$A = 1 - \frac{12}{15}$

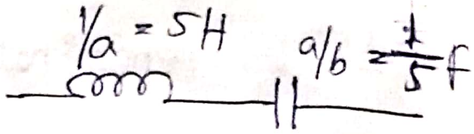
$A = \frac{15-12}{15} = \frac{3}{15} = \frac{1}{5}$

$\therefore \boxed{A = \frac{1}{5}}$

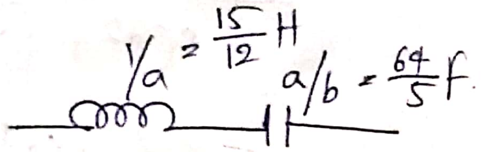
(S4):-

$$Y(s) = \frac{1/5\, s}{s^2+1} + \frac{12/15\, s}{s^2+16}$$

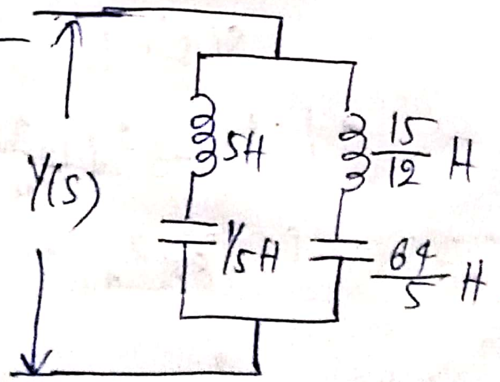
$$Y(s) = \frac{1/5\, s}{s^2+1} = \frac{as}{s^2+b}.$$




$$Y(s) = \frac{12/15\, s}{s^2+16} = \frac{as}{s^2+b}$$




(S5):-



Q. synthesize the given Transfer function using Cauer - I & Cauer - II methods.

$$Z(s) = \frac{2(s+1)(s+3)}{(s+2)(s+6)}.$$

sol: order of numerator = order of denominator.

synthesis will carry with case (2).

Cauer - I :-

Step 1 :- poles :- equate denominator to zero, $(s+2)(s+6) = 0$

$$s = -2, -6.$$

Zeros :- Equate num = 0, $(s+1)(s+3) = 0$

$$s = -1, -3.$$

$s = -1$ i.e, zero is close to origin.

Step-2 :- As zero is close to origin consider $Y(s)$

Given is $Z(s) = \frac{2(s+1)(s+3)}{(s+2)(s+6)}$

but $Y(s) = \frac{1}{Z(s)} = \frac{(s+2)(s+6)}{2(s+1)(s+3)}$

$$Y(s) = \frac{s^2+8s+12}{2[s^2+4s+3]}$$

$$Y(s) = \frac{s^2+8s+12}{2s^2+8s+6}$$

**Step-3 :-** CFE

$$2s^2+8s+6\;\big)\; s^2+8s+12 \;\big(\tfrac{1}{2} \rightarrow Q_1$$
$$\underline{-\,s^2 + \tfrac{8}{2}s + \tfrac{6}{2}}$$
$$4s+9\;\big)\; 2s^2+8s+6 \;\big(\tfrac{s}{2} \rightarrow Q_2$$
$$\underline{-\,2s^2 + \tfrac{9s}{2}}$$
$$\tfrac{7s}{2}+6\;\big)\; 4s+9 \;\big(\tfrac{8}{7} \rightarrow Q_3$$
$$\underline{-\,4s + \tfrac{48}{7}}$$
$$\tfrac{15}{7}\;\big)\; \tfrac{7s}{2}+6 \;\big(\tfrac{49s}{30} \rightarrow Q_4$$
$$\underline{-\,\tfrac{7s}{2}}$$
$$6\;\big)\; \tfrac{15}{7} \;\big(\tfrac{5}{14} \rightarrow Q_5$$
$$\underline{-\,\tfrac{15}{7}}$$
$$\times$$

**Step-4 :-** As $Y(s)$ is considered

$Q_1 = Y_1 = \frac{1}{2}$

$Q_2 = Z_2 = \frac{s}{2}$

$Q_3 = Y_3 = \frac{8}{7}$

$Q_4 = Z_4 = \frac{49s}{30}$

$Q_5 = Y_5 = \frac{5}{14}$

**Step-5 :-** Identify impedance elements.

$Y_1 = \frac{1}{2}$ (a) 'a' form w.r.t Y. — $1/a$ resistor $\Rightarrow$ $2\Omega$

$Z_2 = \frac{s}{2} \Rightarrow \frac{1}{2}(s) \Rightarrow$ 'as' form w.r.t Z — $a$ inductor $\Rightarrow$ $\frac{1}{2}H$

$Y_3 = \frac{8}{7}$ (a) 'a' form w.r.t Y — $1/a$ resistor $\Rightarrow$ $\frac{7}{8}\Omega$

$Z_4 = \frac{49}{30}s = as \Rightarrow$ $a$ inductor $\Rightarrow$ $49/30\,H$

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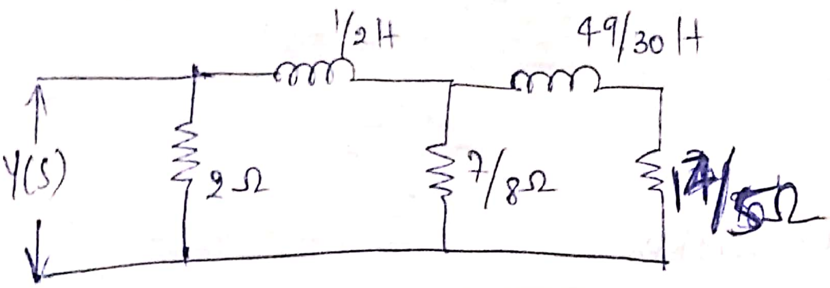
$Y_5 = \frac{15}{4}$ (a) 'a' form w.r.t $Y \Rightarrow$ $1/a$ $\Rightarrow$ $14/5\,\Omega$

step-6 :- As $P_1 = Y_1$

ckt starts shunt impedance.




Cauer - II

Step-1 :- poles: Equate den to zero , $(s+2)(s+6) = 0$

$$s = -2, -6.$$

zeros :- Equate numerator to zero, $(s+1)(s+3) = 0$

$$s = -1, -3.$$

As $s = -1, -2, -3, -6$.

$s = -1$ i.e., zero is close to origin.

Step-2 :- As zero is close to origin Consider $Z(s)$.

Given is $Z(s) = \frac{2(s+1)(s+3)}{(s+2)(s+6)}$

$$Z(s) = \frac{2[s^2+4s+3]}{s^2+8s+12}$$

Arrange order lowest to highest

$$Z(s) = \frac{6+8s+2s^2}{12+8s+s^2}$$

Step-3 :- CFE.

$$12+8s+s^2 \,\big)\, 6+8s+2s^2 \,\big(\, \tfrac{1}{2} \to Q_1$$
$$\cancel{6} + \tfrac{\cancel{8}4}{\cancel{2}}s + \tfrac{s^2}{2}$$
$$-\quad -\quad -$$
$$4s + \tfrac{3s^2}{2} \,\big)\, \cancel{12}+8s+s^2 \,\big(\, \tfrac{3}{s} \to Q_2$$
$$\cancel{12} + \tfrac{9s}{2}$$
$$-\quad -$$
$$\tfrac{7s}{2} + s^2 \,\big)\, \cancel{4s} + \tfrac{3s^2}{2} \,\big(\, \tfrac{8}{7} \to Q_3$$
$$\cancel{4s} + \tfrac{8s^2}{7}$$
$$-\quad -$$
$$\tfrac{5}{14}s^2 \,\big)\, \cancel{\tfrac{7s}{2}} + s^2 \,\big(\, \tfrac{49}{5s} \to Q_4$$
$$\cancel{\tfrac{7s}{2}}$$
$$-$$
$$s^2 \,\big)\, \cancel{\tfrac{5}{14}s^2} \,\big(\, \tfrac{5}{14} \to Q_5$$
$$\cancel{\tfrac{5}{14}}s^2$$
$$\text{(0)}$$

Step-4 :- As $Z(s)$ is considered.

$$Q_1 = Z_1 = \frac{1}{2}$$
$$Q_2 = Y_2 = \frac{3}{s}$$
$$Q_3 = Z_3 = \frac{8}{7}$$
$$Q_4 = Y_4 = \frac{49}{5}s$$
$$Q_5 = Z_5 = \frac{5}{14}$$

Step-5 :- Identify Impedance elements.

$Z_1 = \frac{1}{2}$ (a) 'a' form w.r.t $Z$ — a ⇒ $1/2\ \Omega$.

$Y_2 = \frac{3}{s}$ $\left(\frac{a}{s}\right)$. $\frac{a}{s}$ form w.r.t $Y$ — $1/a$ ⇒ $\frac{1}{3}$ H.

$Z_3 = \frac{8}{7}$ (a). 'a' form w.r.t $z$ — a ⇒ $\frac{8}{7}\ \Omega$.

$Y_4 = \frac{49}{5s}$ & '$\frac{as}{s}$' form w.r.t $y$ — $1/a$ ⇒ ~~$1/\,49/5$ F~~ $5/49$ F

$Z_5 = \frac{5}{4}$ 'a' form w.r.t $z$. — a ⇒ $5/4\ \Omega$

Step - 6 :- As $q_1 = Z_1$

ckt starts with shunt impedance.